Xiezhi: Toward Succinct Proofs of Solvency

Youwei Deng[1][0009−0004−6989−9605]⋆ and Jeremy Clark[2][0000−0002−3533−5965]

[1] University of Alberta, Edmonton, Canada
youwei3@ualberta.ca
[2] Concordia University, Montreal, Canada
j.clark@concordia.com

Abstract. A proof of solvency (or proof of reserves) is a zero-knowledge proof conducted by centralized cryptocurrency exchange to offer evidence that the exchange owns enough cryptocurrency to settle each of its users' balances. The proof seeks to reveal nothing about the finances of the exchange or its users, only the fact that it is solvent. The literature has already started to explore how to make proof size and verifier time independent of the number of (i) users on the exchange, and (ii) addresses used by the exchange. We argue there are a few areas of improvement. First, we propose and implement a full end-to-end argument that is fast for the exchange to prove (minutes), small in size (KBs), and fast to verify (seconds). Second, we deal with the natural conflict between Bitcoin and Ethereum's cryptographic setting (`secp256k1`) and more ideal settings for succinctness (*e.g.,* pairing-based cryptography) with a novel mapping approach. Finally, we discuss how to adapt the protocol to the concrete parameters of `bls12-381` (which is relevant because the bit-decomposition of all user balances will exceed the largest root of unity of the curve for even moderately-sized exchanges).

1 Introduction

When the Bitcoin exchange Mt. Gox was declared bankrupt in 2014, a curious fact was reported in the *New York Times*—the missing 744K BTC "had gone unnoticed for years." Academics showed that proofs of solvency can be done by exchanges in strict zero-knowledge [12], and generated a stream of research papers that continues to improve efficiency [7,15,20,16,27,11] and examine the correctness of deployed proofs [8]. We believe we should continue refining these proofs toward practical implementation as they provide meaningful barriers (or friction) to fraud and incompetence by exchanges.

A proof of solvency (or proof of reserves) is a zero-knowledge proof conducted by centralized cryptocurrency exchange (or more generally, any custodian of cryptocurrencies) to offer evidence that the exchange owns enough cryptocurrency to settle each of its user's balances. The zero-knowledge component protects the exchange's proprietary information such as: number of users, balances of individual users, total balance of all users, which cryptocurrency addresses

⋆ The majority of this work was completed at Concordia University.

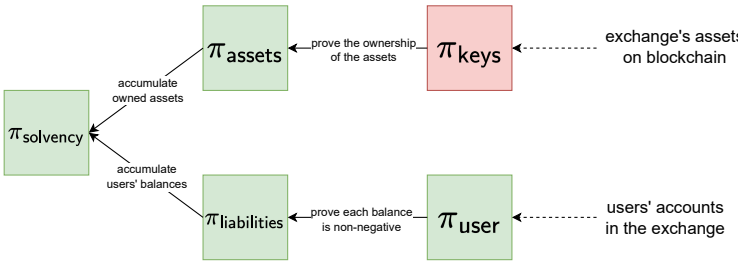


Fig. 1: A high-level overview of Xiezhi and its five sub-components which are coloured green if verifier time and proof size are constant in the number of assets/accounts and red if it is linear. Note π_{keys} is not succinct but the proof of ownership can be reused in different cycles of proving.

belong to the exchange, and total amount of cryptocurrency owned by the exchange. The proof itself is broken into sub-components: (π_{keys}) a proof of knowledge of private signing keys associated with public cryptocurrency addresses (hidden in a freely-composable anonymity set of addresses not belonging to the exchange); (π_{assets}) a summation of these assets into the total assets; (π_{user}) an individualized proof given to each user asserting their balance as used in the overall proof; ($\pi_{\mathsf{liabilities}}$) a summation of these individual liabilities into the total liabilities; and (π_{solvency}) a demonstration that the subtraction of total liabilities from the total assets is at least 0.

Our contributions can be summarized as:

- Xiezhi:[3] A novel and mostly succinct (constant size and verifier time) proof of solvency protocol that covers every step of the "proof[4]", where each sub-component of the proof works with each other sub-component. (Fig. 1)
- A novel technique for mapping knowledge of private keys of common blockchains, such as Bitcoin and Ethereum, from their group (`secp256k1`) into a pairing-friendly group (`bls12-381`) used for succinct arguments.
- Practical adjustments to the protocol to account for concrete parameters, such as the maximum root of unity in `bls12-381`. This facilitates the succinct range proofs for multiple integers.
- Proof of concept implementation of Xiezhi with performance experimentation.

[3] Folklore creature revered in ancient Chinese culture for its ability to distinguish truth from deceit.

[4] We abuse terminology and generally do not distinguish between 'proofs' and 'arguments,' using the term 'proofs' for both. Proofs provide soundness against unbounded malicious provers, while arguments provide zero knowledge against unbounded malicious verifiers. Xiezhi is a hybrid.

Σ-Protocols	The first proofs of solvency are based on Σ-Protocols which work on standard elliptic curves like `secp256k1` but are not succinct (linear proof space, linear verifier time, heavy constants).	Provisions [12]
Inner-product arguments	Protocols like bulletproofs work on standard elliptic curves like `secp256k1` and can reduce some sub-routines (e.g., range arguments) to constant space and logarithmic verifier time.	Provisions with Bulletproofs [7]
Liabilities only	As it is the asset-side of solvency that ties the protocol to standard elliptic curves like `secp256k1`, proving only the liability side can be done in any cryptographic setting.	ZeroLedge [15], DAPOL+ [20], SSVT-based [16], Notus [27], SafeCex[6]
Publish assets	A trivial proof of assets is one that is not zero-knowledge. An exchange could reveal all its addresses and prove ownership by signing a proof-specific message from each address.	Summa[7]
Circuit-level	A general zk-snark can implement any arithmetic circuit, including `secp256k1` operations, which offers a proof of constant size and constant verifier time.	IZPR [11], Proven.tools[8]
MPC in the Head	If the knowledge of some `secp256k1` private keys in an anonymity set can be proved through MPC in the head, a proof of assets can be achieved.	gOTzilla [3]
Custom blockchain	If new blockchains are deployed, they could use digital signatures over pairing-friendly curves.	Mina[9]
Mapping between groups + PIOP	If `secp256k1` values can be mapped to a pairing-friendly group, custom PIOP arguments can potentially reduce the rest of the proof to constant size and constant verifier time.	COPZ [9], Xiezhi

Table 1: How to deal with the fact that Bitcoin and Ethereum use `secp256k1` digital signatures when trying to make a succinct proof of solvency.

Xiezhi has certain limitations: it relies on a universal (shareable with other zk-SNARK systems) trusted setup, secure with one honest participant in a decentralized computation of it [25]; it assumes the public key (as opposed to only its hash) associated with every address in an anonymity set of keys is known (achievable by spending performing at least one transaction)[5]; it is limited to funds controlled by a single public key. That said, it works for any token controlled by public keys with known balances on any such chain or layer 2. Proofs of solvency, generally, also have common limitations shared by Xiezhi: users need to check their balances in the proof, proofs complicate the cover-up of hacks and exit scams but do not prevent them, asset input to the proof can belong to colluders, and Trusted execution environments (TEEs) can help share proving ability without the keys themselves (but Xiezhi's π_{keys} can be adapted for complete knowledge [22]).

[5] We demonstrate Xiezhi with the public keys for brevity. However, Xiezhi can support interacting with hashed keys as well by simply integrating the scheme from Agrawal *et al.* [1]. For details see Appendix D.

2 Related Work

Table 1 reviews research on proofs of solvency, noting that the vast majority of work on proofs of solvency have not attempted an end-to-end proof, focusing instead on just the liabilities or just the assets. Why? We hypothesize that the biggest impediment is that Bitcoin and Ethereum assets are controlled by `secp256k1` private keys. Outside of Bulletproofs (based on inner-product arguments that do not require bilinear pairings and thus, can be implemented in `secp256k1`), most other approaches to succinctness require a specific cryptographic setting that is not `secp256k1` (*i.e.,* RSA for accumulators, pairing-based cryptography for zk-SNARKs, and lattices for zk-STARKs). If one only considers liabilities, then this problem does not have to be dealt with.

3 Preliminaries

3.1 Notation

Our protocols work on a finite field of prime order and are between a prover $\mathcal{P}$ and verifier $\mathcal{V}$. Let $g_{\texttt{secp}}$ and $h_{\texttt{secp}}$ denote two independent generators in the `secp256k1` group $\mathbb{G}_{\texttt{secp}}$; and we use $g_{\texttt{bls}}$ and $h_{\texttt{bls}}$ for `bls12-381`. We denote by $e([x]_1, [y]_2)$ a non-degenerate bilinear pairing, with input groups specified $[x]_i := g_i^x \in \mathbb{G}_i$ for $i = 1, 2$ (when omitted, values are assumed to be in the first group). For vectors, we use an overhead bar to denote a vector and brackets to denote the elements in this vector, e.g., $\overline{v} = \langle v_1, v_2, \ldots \rangle$. Let $\mathbf{C}(x)$ denote a Pedersen commitment to a value x with a hiding factor. Particularly, we use $\mathbf{C}_{\texttt{secp}}$ and $\mathbf{C}_{\texttt{bls}}$ to denote a commitment in `secp256k1` and `bls12-381`, respectively. To help distinguish polynomials and constants, we denote by $\mathcal{C}_f$ a polynomial commitment to f in `bls12-381`. We use ω to denote roots of unity of a field H.

3.2 Terminology

A balance sheet consists of liabilities (value owed to others) and assets (value owned). When the total asset value is the same or more than the total liabilities, the firm is called solvent. The amount by which the assets exceed the liabilities is called capital or equity (depending on context). Some literature prefers the term 'proof of reserves' to 'proof of solvency.'

3.3 zk-SNARK Customization

Modern SNARK toolkits let protocol designers choose how 'close to the metal' they want to work. At the highest level, they can write ordinary software and

[6] V. Buterin, "Having a safe CEX: proof of solvency and beyond," vitalik.ca, 2022
[7] `https://summa.gitbook.io/summa`
[8] `https://www.proven.tools/`
[9] `https://minaprotocol.com/`

prove its execution inside a zkVM. A step lower, they can write a dedicated circuit (or circuit DSL), skipping the zkVM compilation layer for better performance. Going deeper still, they can design the arithmetization directly. We call this *application-specific arithmetization*: specifying a set of vanishing polynomials that capture the protocol semantics. This arithmetization is then instantiated as a succinct proof via a polynomial interactive proof (described below). Xiezhi operates at this layer: we directly engineer the protocol with custom arithmetization. We entirely shortcut the overhead of relying on VMs or circuits.

Circuit-based solutions are feasible but expensive for the prover—the authors of IZPR report about 500K constraints needed per key and proving times in the order of days for an anonymity set of 6000 keys [11]. By contrast, Xiezhi is a few minutes of work for the prover for 6000 keys. In both cases, IZPR and Xiezhi, this step does not need to be repeated often, only when the exchange wants to introduce new keys holding its assets. It is also important to recognize IZPR can let the exchange add keys it has not used yet to π_{keys}, further reducing how often this proof needs to be redone. This is a desirable property we are not able to easily achieve in Xiezhi (in short, it is due to our use of selector polynomials instead of lookup arguments but future work could explore blending the best properties of Xiezhi and IZPR).

3.4 Cryptographic Building Blocks

We refer the reader to Appendix A for the following cryptographic primitives: discrete logarithm assumption (Section A.1), Pedersen commitments (Section A.2), zero-knowledge proof and its related properties (Section A.3), Σ-protocols (conjunction and disjunction) (Section A.4,A.5), and roots of unity (Section A.6).

KZG Evaluation Variants: We assume the readers are familiar with polynomial commitment schemes. Briefly speaking, a polynomial commitment scheme (PCS) enables a succinct demonstration that a commitment $\mathcal{C}$ to a polynomial f is correct with perfect hiding and computational binding. We use the KZG PCS [21] in its zero-knowledge form, which requires (i) randomized commitments and (ii) committing to additional random points, so checks of polynomial relationships at random points do not leak information about the polynomial. In this paper, we refer to such KZG variant as KZG_{zk}. In Section A.7, we specify KZG_{zk} and prove it is complete, sound, and HVZK.

Moreover, it is possible to 'stop early' in the KZG commitment scheme and end up with a Pedersen commitment to $f(a)$, instead of revealing $f(a)$ in plaintext [2]. We call this 'open to a commitment' variant (KZG_{cm}) with two algorithms, $\mathbf{C}_{\mathtt{bls}}(f(\zeta)) \leftarrow \mathsf{KZG}_{cm}.\mathsf{Prove}(f; f(\zeta); \zeta)$ to open the committed evaluation of $f(\zeta)$ in `bls12-381`, and $\{1/0\} \leftarrow \mathsf{KZG}_{cm}.\mathsf{Verify}(\mathcal{C}_f; \mathbf{C}_{\mathtt{bls}}(f(\zeta)); \zeta)$ to verify the committed value is correct. In Section A.9, we specify KZG_{cm}.

Polynomial Interactive Oracle Proof (PIOP): Gabizon *et al.* [19] popularized the definition of a universal polynomial protocol, a.k.a polynomial interactive oracle proof (PIOP). A PIOP enables $\mathcal{P}$ to prove the correctness of the

claimed relations among polynomials without revealing the polynomials. It particularly requires a (PCS) to be achieved. Our work integrates the above variant of the KZG commitment scheme into the PIOP. We refer to Section A.10, A.11 for the definitions of polynomial relation and PIOP. We use $\texttt{PIOP}(\mathcal{R}_1, \mathcal{R}_2, \dots)$ to denote the polynomial protocol that $\mathcal{P}$ wants to prove each relation $\mathcal{R}_i$ holds for the prescribed polynomials.

Range Proof Variant: A zero-knowledge range proof (ZKRP) enables $\mathcal{P}$ to convince $\mathcal{V}$ a value x is in a specified range, e.g., $[0, 2^k)$, without revealing x. ZKRPs have three typical approaches: square decomposition, n-ary decomposition, and hash chains [10]. We use the succinct decomposition method from Boneh *et al.* [6](Section A.8). However, we adapt it to prove multiple values are in the specified range. We will introduce and motivate our variant in Section 5.1.

4 Proof of Assets (PoA)

We are now ready to explain Xiezhi. We will begin on the asset side, taking the example of ETH. The PoA is broken into two steps: π_{keys} and π_{assets}. In short, for π_{keys}, the exchange publishes a public vector of Ethereum public keys, consisting of its own public keys hidden amongst a larger anonymity set of keys from the blockchain [10]. Next, it will output a hiding commitment to a binary 'selector' vector (in `bls12-381`) and prove it records a 0 if the exchange is not claiming to know the secret key (in `secp256k1`) of the public key in the same position in the vector, and a 1 if the exchange can prove knowledge of the secret key. For π_{assets}, the exchange publishes a public vector of ETH balances matched to the vector of public keys. It outputs a hiding commitment to the sum of its assets, as the first element of a vector, and proves it is the sum of the subset of balances in the balance vector marked by the selector vector.

4.1 The π_{keys} Proof

Before presenting our π_{keys} proof, we quickly discuss a few approaches that helped us develop it. A highly relevant Σ-protocol from the literature, COPZ, proves that two commitments in two different groups (*e.g.,* `secp256k1` and `bls12-381`) commit to the same value [9]. The paper cites proof of assets as a use-case but does not work out a protocol. COPZ allows an exchange to 'map' private keys from `secp256k1` to `bls12-381`. Two places this mapping could occur would be at the very start or the very end of the assets proof. At the end, it might look like this: an existing proof of assets protocol (*e.g.,* Provisions [12]) could be run to create a commitment to the total assets in `secp256k1`, then COPZ can be used to prove the same commitment in `bls12-381`, and finally this can be 'glued' to

[10] The anonymity set should be static over time. The exchange can add more keys to the set but should not remove the existing keys, otherwise π_{keys} has to be performed again for a new round of proofs of solvency.

$\mathcal{P}$ and $\mathcal{V}$ are both given $\{\mathbf{C}_{\texttt{bls}}(s_i)\}$ where s_i is the i-th element in the selector vector $\overline{s} = \langle s_1, s_2, \ldots, s_\kappa \rangle$. $\mathcal{P}$ has the access to $\{\mathsf{sk}_i\}, \overline{s}$, and the hiding factor of $\mathbf{C}_{\texttt{bls}}(s_i)$, r_i.

1. Case 1: $s_i = 1$ ($\mathcal{P}$ claims knowledge of sk_i)
   (a) $\mathcal{P}$ selects $e_1 \xleftarrow{\$} \{0,1\}^t; z_3, \beta \xleftarrow{\$} \mathbb{Z}_{\texttt{bls}}; \alpha \xleftarrow{\$} \mathbb{Z}_{\texttt{secp}}$
   (b) $\mathcal{P}$ publishes $t_1 = g_{\texttt{secp}}^{\alpha}$
   (c) $\mathcal{P}$ publishes $t_2 = h_{\texttt{bls}}^{\beta}$
   (d) $\mathcal{P}$ publishes $t_3 = g_{\texttt{bls}}^{-e_1} h_{\texttt{bls}}^{z_3 - r_i e_1}$
   (e) $\mathcal{V}$ publishes t-bit challenge $e \xleftarrow{\$} \{0,1\}^t$ (or $\mathcal{P}$ via Fiat-Shamir)
   (f) $\mathcal{P}$ computes $e_0 = e \oplus e_1$ and publishes e_0 and e_1
   (g) $\mathcal{P}$ publishes $z_1 = e_0 \mathsf{sk}_i + \alpha$
   (h) $\mathcal{P}$ publishes $z_2 = e_0 r_i + \beta$
   (i) $\mathcal{P}$ publishes z_3
2. Case 2: $s_i = 0$ ($\mathcal{P}$ does not claim knowledge of sk_i)
   (a) $\mathcal{P}$ selects $e_0 \xleftarrow{\$} \{0,1\}^t; z_1 \xleftarrow{\$} \mathbb{Z}_{\texttt{secp}}; z_2, \alpha \xleftarrow{\$} \mathbb{Z}_{\texttt{bls}}$
   (b) $\mathcal{P}$ publishes $t_1 = g_{\texttt{secp}}^{z_1} / \mathsf{pk}_i^{e_0}$
   (c) $\mathcal{P}$ publishes $t_2 = g_{\texttt{bls}}^{e_0} h_{\texttt{bls}}^{z_2 - r_i e_0}$
   (d) $\mathcal{P}$ publishes $t_3 = h_{\texttt{bls}}^{\alpha}$
   (e) $\mathcal{V}$ publishes t-bit challenge $e \xleftarrow{\$} \{0,1\}^t$ (or $\mathcal{P}$ via Fiat-Shamir)
   (f) $\mathcal{P}$ computes $e_1 = e \oplus e_0$ and publishes e_0 and e_1
   (g) $\mathcal{P}$ publishes z_1
   (h) $\mathcal{P}$ publishes z_2
   (i) $\mathcal{P}$ publishes $z_3 = e_1 r_i + \alpha$
3. $\mathcal{V}$ outputs **acc** if and only if
   (a) $e = e_0 \oplus e_1$
   (b) $g_{\texttt{secp}}^{z_1} = \mathsf{pk}_i^{e_0} t_1$
   (c) $g_{\texttt{bls}}^{e_0} h_{\texttt{bls}}^{z_2} = \mathbf{C}_{\texttt{bls}}(s_i)^{e_0} t_2$
   (d) $h_{\texttt{bls}}^{z_3} = \mathbf{C}_{\texttt{bls}}(s_i)^{e_1} t_3$

Protocol 1: The ZKPoK proof demonstrates that $\mathcal{P}$ can prove knowledge of a secret key with the correct committed selector.

a succinct proof of liabilities in `bls12-381`. However, this does not leverage the fact that `bls12-381` might help make the assets proof succinct.

Alternatively, the exchange can map all their keys at the start. There is a roadblock: the exchange can only map keys they know the secret key for and the exchange cannot reveal which keys they know and which they do not. Assume there is a protocol that would allow the exchange to output a sparse vector of `bls12-381` private key values (sorted by index of known ETH public keys) containing the key value when they know it, and recording a 0 if they do not. We designed such a protocol only to realize that the key values in `bls12-381` are actually never used, we only use the fact that knowledge of them is proven (which is covered by the ability to produce the value in `bls12-381`) and the fact that unclaimed keys are zeroed-out.

This leads to our key observation: we do not need to map *values* from `secp256k1` to `bls12-381`, we just have to map the *success or failure* of a ZKP in `secp256k1` to `bls12-381`. This can be accomplished by composing (conjunction and disjunction of) Σ-protocols. The prover (exchange) will choose a set of Ethereum public keys as its anonymity set of size κ (containing its actual keys) $\{\mathsf{pk}_1, \mathsf{pk}_2, \ldots, \mathsf{pk}_\kappa\}$ and publish them in an ordered (indexed) way. It will also create a binary 'selector' vector $\overline{s} = \langle s_1, s_2, \ldots, s_\kappa \rangle$ with a 1 in the same index of every key it is claiming to know and a 0 in the index of the keys it does not know (or does not want to claim for whatever reason). This vector is interpolated into a polynomial $f_{\mathsf{keys}}(X)$ as the evaluation points and committed to $\mathbf{C}_{\mathtt{bls}}(f_{\mathsf{keys}}(X))$ using the KZG polynomial commitment scheme [21]. For each index i, the prover shows the evaluation of $f_{\mathsf{keys}}(X_i)$ but instead of providing the evaluation value s_i in plaintext, it provides a Pedersen commitment to it $\mathbf{C}_{\mathtt{bls}}(s_i)$ (a mild modification of the KZG showing protocol detailed in Section A.9). It then shows the value is correct with the following Σ-protocol (for details see Protocol 1):

$$\text{ZKPoK}\{(\mathsf{sk}_i, s_i) : [\mathsf{pk}_i = g_{\mathtt{secp}}^{\mathsf{sk}_i} \wedge \mathbf{C}_{\mathtt{bls}}(s_i) = \mathbf{C}_{\mathtt{bls}}(1)] \vee \mathbf{C}_{\mathtt{bls}}(s_i) = \mathbf{C}_{\mathtt{bls}}(0)\}$$

In plain English, the prover either: (1) puts a 0 in the selector vector; or (2a) puts a 1 in the selection vector *and* (2b) knows the private key of the given public key. (1) and (2a) are a PoK of a representation for Pedersen commitments in `bls12-381` while (2b) is a Schnorr PoK of a discrete logarithm in `secp256k1`—both well studied Σ-protocols [13,24]. The fact that (1) and (2a) are in `bls12-381` while (2b) is in `secp256k1` is not problematic because the disjunction ($\vee$) and conjunction ($\wedge$) operations for composing Σ-protocols are based only on how challenge values are constructed and both groups (`secp256k1` and `bls12-381`) can encode a large t-bit challenge (*e.g.,* $t = 254$) into their exponent groups.

As this protocol is repeated for each key, it is not succinct and will be linear in proof size and verifier time. However, once the selector array is proven correct, the exchange can re-use it every time it does a proof of solvency until it updates its keys. The full details are provided in Protocol 2.

4.2 The π_{assets} Argument

The π_{keys} protocol proves that $\mathcal{C}_{f_{\mathsf{sel}}}$ is a commitment to a selector polynomial $f_{\mathsf{sel}}(X)$ (in `bls12-381`) which marks the public keys owned by the exchange. At a given time (block number), the balances of every public key included in the anonymity set will be encoded in polynomial $f_{\mathsf{bal}}(X)$. The product of $f_{\mathsf{sel}}(X) \cdot f_{\mathsf{bal}}(X)$ will preserve the balance values owned by the exchange and zero-out the balance values not claimed by the exchange. The final step is producing a summation over the values in $f_{\mathsf{sel}}(X) \cdot f_{\mathsf{bal}}(X)$. The exchange will put $f_{\mathsf{sel}}(X) \cdot f_{\mathsf{bal}}(X)$ in accumulator form $f_{\mathsf{assets}}(X)$ and prove its correctness. In this form, the total assets will sit at the head (first index) of $f_{\mathsf{assets}}(X)$, which is $f_{\mathsf{assets}}(\omega^0)$. The full details are provided in Protocol 3.

1. $\mathcal{P}$ publishes an anonymity set of public keys: $\langle \mathsf{pub\_key}_1, \mathsf{pub\_key}_2, \mathsf{pub\_key}_3, \ldots, \mathsf{pub\_key}_\kappa \rangle$.
2. $\mathcal{P}$ constructs a selector vector: $\overline{s} = \langle s_1, s_2, s_3, \ldots, s_\kappa \rangle$ where $s_i = 0$ unless $\mathcal{P}$ can prove knowledge of sk_i given $\mathsf{pub\_key}_i = g_{\mathsf{secp}}^{\mathsf{sk}_i}$, then $s_i = 1$.
3. $\mathcal{P}$ interpolates a polynomial $f_{\mathsf{sel}}(X)$ from $\overline{s}$ and commits to f_{sel}.
4. For each i, $\mathcal{P}$ computes $\mathbf{C}_{\mathtt{bls}}(s_i) \leftarrow \mathsf{KZG}_{cm}.\mathsf{Prove}(f_{\mathsf{sel}}; s_i; \omega^i)$.
5. For each $\mathbf{C}_{\mathtt{bls}}(s_i)$
   (a) $\mathcal{P}$ and $\mathcal{V}$ run Protocol 1 to prove ZKPoK$\{(\mathsf{sk}_i, s_i) : [\mathsf{pub_key}_i = g_{\mathsf{secp}}^{\mathsf{sk}_i} \wedge \mathbf{C}_{\mathtt{bls}}(s_i) = \mathbf{C}_{\mathtt{bls}}(1)] \vee \mathbf{C}_{\mathtt{bls}}(s_i) = \mathbf{C}_{\mathtt{bls}}(0)\}$.
   (b) $\mathcal{V}$ aborts if $\mathsf{KZG}_{cm}.\mathsf{Verify}(\mathcal{C}_{f_{\mathsf{sel}}}; \mathbf{C}_{\mathtt{bls}}(s_i); \omega^i)$ returns 0.

Protocol 2: The π_{keys} proof demonstrates that $f_{\mathsf{sel}}(X)$ encodes a binary selector vector of the public keys for which the exchange can prove knowledge of the corresponding secret key.

– Inputs
  - $\mathtt{pk} := f_{\mathsf{bal}}(X)$, where f_{bal} is interpolated from the balances $\langle \mathtt{bal}_1, \mathtt{bal}_2, \ldots, \mathtt{bal}_\kappa \rangle$ of each public key in the anonymity set
  - $\mathtt{vk} := \mathcal{C}_{f_{\mathsf{bal}}}$
– Protocol
  1. $\mathcal{P}$ takes as input the selector vector of keys shown ownership of, $f_{\mathsf{sel}}(X)$, from π_{keys}.
  2. $\mathcal{P}$ constructs an accumulative polynomial $f_{\mathsf{assets}}(X)$ such that
     (a) $f_{\mathsf{assets}}(X) - f_{\mathsf{assets}}(X\omega) = f_{\mathsf{bal}}(X) \cdot f_{\mathsf{sel}}(X), X \neq \omega^{\kappa-1}$
     (b) $f_{\mathsf{assets}}(X) = f_{\mathsf{bal}}(X) \cdot f_{\mathsf{sel}}(X), X = \omega^{\kappa-1}$
  3. $\mathcal{P}$ commits to $f_{\mathsf{assets}}, f_{\mathsf{bal}}, f_{\mathsf{sel}}$ and sends the commitments to $\mathcal{V}$.
  4. $\mathcal{P}$ opens the committed evaluation of f_{assets} at ω^0, i.e., $\mathbf{C}_{\mathtt{bls}}(f_{\mathsf{assets}}(\omega^0)) \leftarrow \mathsf{KZG}_{cm}.\mathsf{Prove}(f_{\mathsf{assets}}; f_{\mathsf{assets}}(\omega^0); \omega^0)$.
  5. $\mathcal{P}$ and $\mathcal{V}$ run $\mathtt{PIOP}$ to prove the following relations hold over $H_{\mathsf{assets}} = \{\omega^0, \omega^1, \ldots, \omega^{\kappa-1}\}$

$$\mathcal{R}_1 = \{f_{\mathsf{assets}}, f_{\mathsf{bal}}, f_{\mathsf{sel}} \mid [f_{\mathsf{assets}}(X) - f_{\mathsf{assets}}(X\omega) - f_{\mathsf{bal}}(X) \cdot f_{\mathsf{sel}}(X)] \cdot (X - \omega^{\kappa-1}) = 0\}$$

$$\mathcal{R}_2 = \left\{ f_{\mathsf{assets}}, f_{\mathsf{bal}}, f_{\mathsf{sel}} \;\middle|\; [f_{\mathsf{assets}}(X) - f_{\mathsf{bal}}(X) \cdot f_{\mathsf{sel}}(X)] \cdot \frac{X^\kappa - 1}{X - \omega^{\kappa-1}} = 0 \right\}$$

  6. $\mathcal{V}$ outputs **acc** if and only if
     (a) The above relations hold.
     (b) $\mathsf{KZG}_{cm}.\mathsf{Verify}(\mathcal{C}_{f_{\mathsf{assets}}}; \mathbf{C}_{\mathtt{bls}}(f_{\mathsf{assets}}(\omega^0)); \omega^0)$ return 1.

Protocol 3: The π_{assets} proof demonstrates that the balances associated with each key in the anonymity set are included, the subset not owned by the exchange (per selector vector from π_{keys}) are zero-ed out, and remaining balances are totalled correctly in $f_{\mathsf{assets}}(\omega^0)$.

5 Proof of Liabilities

5.1 The $\pi_{\mathsf{liabilities}}$ Argument

The exchange will commit to every user balance and produce a commitment of the total amount across all balances. Since the exchange is free to make-up additional users and include them, we want to make sure this does not help an insolvent exchange in any way. To do this, we force all balances to be zero or positive numbers. For a finite field, this means small integers that have no chance of exceeding the group order (modular reduction) when added together. In practice, we can limit ourselves to an even smaller range that is sufficient to capture what a balance in ETH (or fractions of ETH) might look like. These balances are expressed in binary and we use range proof from Section A.8.

However, when we turn to implement this in practice, we encounter a roadblock. If μ balances are placed as k-bit numbers side-by-side in a vector, we need a vector of size $\mu \cdot k$. If we want to optimize polynomial interpolation, we encode our array at x-coordinates that correspond to the roots of unity of the exponent group (see Section A.6) and for `bls12-381`, we can only efficiently encode data vectors of length up to $2^{32} = 4,294,967,296$.[11] Consider an exchange with $\mu = 1,000,000$ accounts, only 12 bits are left to capture account balances, say, as between 0.01 and 40.96 ETH ($30 to $150K USD at time of writing). Exchanges could have more than 1 million accounts, the largest could be more than $150K USD, and an exchange could have a long tail of accounts with balances less than $30 such that rounding them all up to $30 creates a solvency issue. Clearly $k = 2^{32}$ is not large enough for directly encoding liabilities (as binary numbers) into a single polynomial.

To deal with this issue, there are three main alternatives. (1) The exchange can encode points at arbitrary x-coordinates and use general (Laplacian) interpolation, (2) the exchange can break down what it is proving into chunks but this will require one succinct proof per chunk, or (3) the range proof could be adapted for decomposition into something larger than bits (*e.g.,* bytes or 32-bit words). The latter may be feasible with lookup arguments, but we do not pursue modifying the range proof [6] in this work. Instead we opt for (2). Specifically we will produce a polynomial argument for the first bit of every account, for the second bit of every account, *etc.* This means $\pi_{\mathsf{liabilities}}$ will be linear in proof size and verifier work but it is linear in the bit-precision of each account (k) and is in fact constant (succinct) in terms of the number of users on the exchange. For example, we will later show if accounts are captured with a precision of 32-bits, the proof size will be under 10KB and verifier time will be under 8ms independent of the number of users on the exchange (see Figure 4).

The protocol creates k polynomials—the kth polynomial p_k for the last bit of each of the μ accounts, the last second polynomial for the last second bit of every account, *etc.* It conducts a range proof 'vertically' (across $\{p_1(\omega^i), p_2(\omega^i), \ldots, p_k(\omega^i)\}$ for $\mathfrak{bal}_i$) for each account (for all i). It then converts the bits into integers 'vertically' ($p_j(\omega^i) - 2p_{j+1}(\omega^i) \in \{0,1\}, j \in [1,k)$, so

[11] The exponent group in `bls12-381` has 2-adicity of 32.

– Inputs
  - $\texttt{pk} := \{x_1, x_2, \ldots, x_\mu\}$, the values to be proved in $[0, 2^k)$
– Protocol
  1. $\mathcal{P}$ computes the binary decomposition (from most significant bit to least significant bit) of each balance, $\{z_j^{(x_i)}\}_{i\in[\mu],j\in[k]}$, such that $z_j^{(x_i)} \in \{0,1\}$ and $x_i = \sum_{j=1}^{k} 2^{j-1} \cdot z_j^{(x_i)}$.
  2. $\mathcal{P}$ puts the bits into accumulator form where $\chi_k^{(x_i)} = z_k^{(x_i)}$ and $\chi_i^{(x_i)} = 2\chi_{i+1}^{(x_i)} + z_i^{(x_i)}$. (Remark: visualized as a matrix, each row is a balance where the k-th column is the least significant bit and, moving right-to-left, each bit is folded in until it accumulates to x_j in the first column.)

$$\begin{bmatrix} \chi_1^{(x_1)} & \chi_2^{(x_1)} & \chi_3^{(x_1)} & \cdots & \chi_k^{(x_1)} \\ \chi_1^{(x_2)} & \chi_2^{(x_2)} & \chi_3^{(x_2)} & \cdots & \chi_k^{(x_2)} \\ \chi_1^{(x_3)} & \chi_2^{(x_3)} & \chi_3^{(x_3)} & \cdots & \chi_k^{(x_3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \chi_1^{(x_\mu)} & \chi_2^{(x_\mu)} & \chi_3^{(x_\mu)} & \cdots & \chi_k^{(x_\mu)} \end{bmatrix}$$

  3. Due to the concrete parameters of `bls12-381`, $\mathcal{P}$ will work column-by-column (proof size and verifier time will be linear in k which is the bit-precision of each account). Let column j be vector $\overline{p}_j = \{\chi_j^{(x_1)}, \chi_j^{(x_2)}, \ldots, \chi_j^{(x_\mu)}\}$. The following constraints apply (for $i \in [\mu], j \in [1,k)$): $\overline{p}_1[i] = x_i$; $\overline{p}_j[i] - 2 \cdot \overline{p}_{j+1}[i] \in \{0,1\}$; and $\overline{p}_k[i] \in \{0,1\}$. $\overline{p}_1$ contains $\{x_1, x_2, \ldots, x_\mu\}$.
  4. $\mathcal{P}$ interpolates polynomials for $\overline{p}_j \to p_j(X)$ and publishes commitments to each.
  5. $\mathcal{P}$ and $\mathcal{V}$ run `PIOP` to prove the following relations over H_{rp} where $H_{\mathsf{rp}} = \{\omega^0, \omega^1, \ldots, \omega^{k-1}\}$

$$\begin{aligned} \mathcal{R}_1 &= \{p_1, p_2 \mid [p_1(X) - 2p_2(X)] \cdot [1 - (p_1(X) - 2p_2(X))] = 0\} \\ \mathcal{R}_2 &= \{p_2, p_3 \mid [p_2(X) - 2p_3(X)] \cdot [1 - (p_2(X) - 2p_3(X))] = 0\} \\ &\vdots \\ \mathcal{R}_{k-1} &= \{p_{k-1}, p_k \mid [p_{k-1}(X) - 2p_k(X)] \cdot [1 - (p_{k-1}(X) - 2p_k(X))] = 0\} \\ \mathcal{R}_k &= \{p_k \mid p_k(X) \cdot [1 - p_k(X) = 0\} \end{aligned}$$

  6. $\mathcal{V}$ outputs **acc** if and only if the above relations hold.

Protocol 4: The range proof for multiple values demonstrates that each value is either zero or a positive number less than a specified value 2^k.

that $p_1(\omega^i) = \mathfrak{bal}_i$) for each account, creating a polynomial f_{liab} of each user's balance. Last it sums up all elements 'horizontally' ($\sum_{i=1}^{\mu-1} f_{\mathsf{liab}}(\omega^i)$) in f_{liab} to produce the total liabilities (Protocol 4). The bit decomposition is argued with

- Inputs
  - $\mathtt{pk} := \{p_1, p_2, \ldots, p_k\}$, which comes from running Protocol 4 on the liabilities $\{\mathfrak{bal}_1, \mathfrak{bal}_2, \ldots, \mathfrak{bal}_\mu\}$
- Protocol
  1. $\mathcal{P}$ builds an additive accumulator $\overline{\nu}$ for p_1 where $\nu_\mu = \mathfrak{bal}_\mu = p_1(\omega^{\mu-1})$ and $\nu_i = \nu_{i+1} + \mathfrak{bal}_i, i \in [1, \mu]$. Remark: ν_1 will contain the total liability value.
  2. $\mathcal{P}$ interpolates f_{liab} from $\overline{\nu}$ and publishes the commitments to f_{liab} and $\{p_1, p_2, \ldots, p_k\}$.
  3. $\mathcal{P}$ opens the committed evaluation of f_{liab} at ω^0, i.e., $\mathbf{C}_{\mathtt{bls}}(f_{\mathsf{liab}}(\omega^0)) \leftarrow \mathsf{KZG}_{cm}.\mathsf{Prove}(f_{\mathsf{liab}}; f_{\mathsf{liab}}(\omega^0); \omega^0)$
  4. $\mathcal{P}$ and $\mathcal{V}$ run $\mathtt{PIOP}$ to prove the following relations hold over H_{liab} where $H_{\mathsf{liab}} = \{\omega^0, \omega^1, \ldots, \omega^{\mu-1}\}$

$$\mathcal{R}_1 = \left\{ f_{\mathsf{liab}}, p_1 \;\middle|\; [f_{\mathsf{liab}}(X) - f_{\mathsf{liab}}(X\omega) - p_1(X)] \cdot (X - \omega^{\mu-1}) = 0 \right\}$$

$$\mathcal{R}_2 = \left\{ f_{\mathsf{liab}}, p_1 \;\middle|\; [f_{\mathsf{liab}}(X) - p_1(X)] \cdot \frac{X^\mu - 1}{X - \omega^{\mu-1}} = 0 \right\}$$

  5. $\mathcal{V}$ outputs **acc** if and only if
     (a) The above relations hold.
     (b) $\mathsf{KZG}_{cm}.\mathsf{Verify}(\mathcal{C}_{f_{\mathsf{liab}}}; \mathbf{C}_{\mathtt{bls}}(f_{\mathsf{liab}}(\omega^0)); \omega^0)$ return 1.

Protocol 5: The $\pi_{\mathsf{liabilities}}$ proof demonstrates that each liability is either zero or a positive number, and that the balances are totalled correctly in $f_{\mathsf{liab}}(\omega^0)$.

1. $\mathcal{P}$ interpolates the identifier polynomial $f_{\mathsf{uid}}(X)$ such that $f_{\mathsf{uid}}(X_i) = \mathfrak{uid}_i$ for i from 1 to μ.
2. $\mathcal{P}$ publishes the commitments to $f_{\mathsf{uid}}(X)$ and p_1 (from $\pi_{\mathsf{liabilities}}$ above).
3. For check from user i, $\mathcal{P}$ tells the user he is at index i and opens $f_{\mathsf{uid}}(\omega^{i-1})$ and $p_1(\omega^{i-1})$.
4. $\mathcal{V}$ outputs **acc** if and only if
   (a) Its user identifier is the evaluation of f_{uid} at the given point ω^{i-1}.
   (b) Its balance is the evaluation of p_1 at the given point ω^{i-1}.

Protocol 6: The π_{users} proof demonstrates that to each user who checks that their balance is recorded correctly under a unique identifier for them (to mitigate clash attacks).

the range proof and the summation of balances is argued with a sum-check. The full protocol is given in Protocol 5.

5.2 The π_{users} Argument

The π_{users} argument is conducted between the exchange and the user, so the user can check that their balance is correctly encoded into the polynomials used in

1. $\mathcal{P}$ computes equity eq as the total assets minus the total liabilities.
2. $\mathcal{P}$ publishes commitment to polynomial $f_{\mathsf{eq}}(X)$ where $f_{\mathsf{eq}}(\omega^0) = \mathsf{eq}$.
3. $\mathcal{P}$ generates a range proof for eq in $f_{\mathsf{eq}}(X)$ to demonstrate it is a non-negative integer.
4. $\mathcal{P}$ opens $f_{\mathsf{eq}}(\omega^0)$ through KZG_{cm} and publishes $\mathbf{C}_{\mathtt{bls}}(f_{\mathsf{assets}}(\omega^0))$, $\mathbf{C}_{\mathtt{bls}}(f_{\mathsf{liab}}(\omega^0))$ from π_{assets} and $\pi_{\mathsf{liabilities}}$.
5. $\mathcal{V}$ outputs **acc** if and only if
   (a) $\mathbf{C}_{\mathtt{bls}}(f_{\mathsf{assets}}(\omega^0)) = \mathbf{C}_{\mathtt{bls}}(f_{\mathsf{liab}}(\omega^0)) \cdot \mathbf{C}_{\mathtt{bls}}(f_{\mathsf{eq}}(\omega^0))$.
   (b) The range proof for eq is valid.

Protocol 7: The π_{solvency} proof demonstrates that the total assets exceed the total liabilities by a non-negative integer (called the equity).

$\pi_{\mathsf{liabilities}}$. If two users have the same balance, a malicious exchange might include only one of the balances and open up the same balance for each user. Unless the users compared their proofs, they would not catch the exchange (*cf.* [8]). This attack appears in other cryptographic protocols where users need to check things, the main one being cryptographic voting schemes. It has been studied under general definitions as a 'clash attack' [23]. The solution is to label each balance with a unique user identifier [12]. Labeling can be done with an additional polynomial of labels under the assumption that a user ID and a balance need to be at the same index. A user ID can be the hash of the user's account name or email address. The full protocol is given in Protocol 6.

5.3 The π_{solvency} Argument

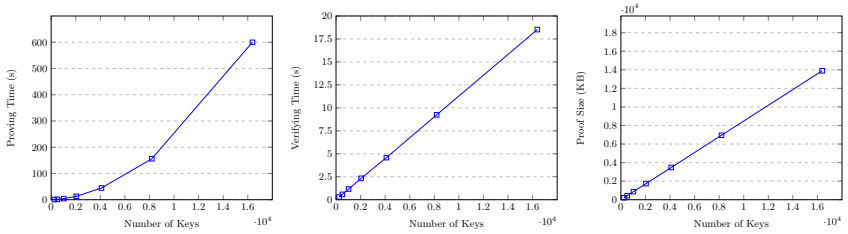
The final step of the proof is prove the total assets exceed the total liabilities. At the end of π_{assets}, the total assets are contained in the polynomial evaluation point $f_{\mathsf{assets}}(\omega^0)$; while at the end of $\pi_{\mathsf{liabilities}}$, the total liabilities are contained in $f_{\mathsf{liab}}(\omega^0)$. Assuming assets exceed liabilities by some amount, this amount can be added to the liability-side to provide a difference of exactly zero. The full argument is given in Protocol 7.

6 Security Analysis

We adapt the security definition of a zero-knowledge proof of solvency from Provisions [12] in Section C.1. We refer to Section C.3 for the proofs of Xiezhi's sub-components: π_{keys}, π_{assets}, $\pi_{\mathsf{liabilities}}$, π_{assets}, π_{users}, and π_{solvency}. Here we state the main theorem:

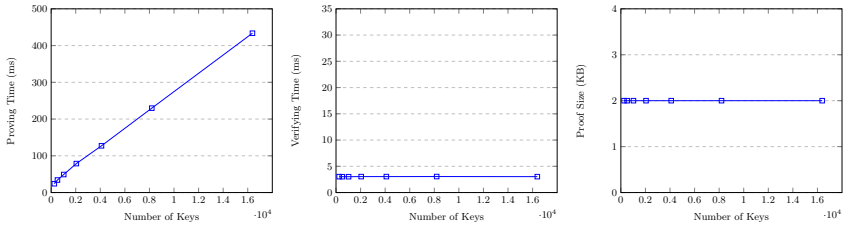
Theorem 1. *Xiezhi (Xiezhi* $\leftarrow \langle \pi_{\mathsf{keys}}, \pi_{\mathsf{liabilities}}, \pi_{\mathsf{assets}}, \pi_{\mathsf{users}} \rangle$*) is a privacy-preserving proof of solvency with respect to Definition 8.*

Proof. See Section C.4.



(a) (b) (c)

Fig. 2: Performance of π_{keys}. Subfigure (a) illustrates the number of keys and the proving time; Subfigure (b) and (c) indicate the verifying time and the proof size are linear in the number of keys.



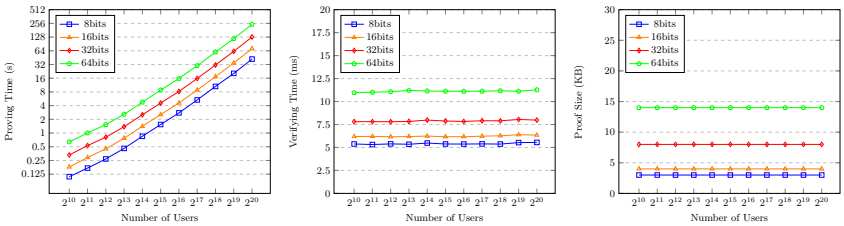
(a) (b) (c)

Fig. 3: Performance of π_{assets}. Subfigure (a) and (b) suggest it takes 433.66 milliseconds to generate the proof and 37.57 milliseconds to verify the proof for 16,384 keys; Subfigure (c) shows the proof size is constant, 2KB, based on our implementation.

7 Performance Evaluation

We analyzed the theoretical performance of Xiezhi and compared it with prior works in Section B.1. We also implemented the concept of Xiezhi in Rust based on the popular library, arkworks[13]. The implementation is publicly accessible on GitHub[14]. Our implementation chose the pairing-friendly elliptic curve `bls12-381` for the KZG commitment which has 128-bit security. The experiments were conducted on a personal computer with i9-13900KF and 32GB of memory. The experimental data including balances and `secp256k1` key pairs are randomly generated locally for simplicity. Since there is no range-proof for PoA, we tested the PoA with balances randomly distributed in $[1, 2^{64})$ to simulate the real distribution of assets, and for PoL, we tested the program with balances randomly distributed in $[1, 2^8)$, $[1, 2^{16})$, $[1, 2^{32})$, and $[1, 2^{64})$. We simulated $2^8, 2^9 \ldots, 2^{14}$ and $2^{10}, 2^{11}, \ldots, 2^{20}$ users for PoA and PoL respectively. Simulating different numbers of users for PoA and PoL is because π_{keys} was

[13] `https://github.com/arkworks-rs`
[14] `https://github.com/Shvier/proof-of-solvency`



(a) (b) (c)

Fig. 4: Performance of $\pi_{\mathsf{liabilities}}$ by different size of the range proof. Our experiments show the proving time grows linearly by the number of users while the verifying time and the proof size are constant when the size of the range proof is fixed. From the test result of Binance's PoL, it needs 1.5 days to generate the proof for 100 million accounts with 100 servers [12], but our PoL requires less than 10 minutes with the same number of servers. This indicates our protocol is practical to handle real-world applications.

time-consuming for a larger number of users. For each protocol, we ran the test ten times with the same experimental data. Our figures are interpolated from the average performance of ten times discarding the maximum and minimum of the samples.

8 Open Research Challenges

The efficiency and succinctness of Xiezhi might be further improved. Recall the heaviest work of π_{keys} is proving each committed point is correct, and the opening scheme we demonstrated from Plonk requires $t \cdot d$ scalar multiplications for prover, where t is the number of the opening points and d is the degree bound of the polynomial. The work in BDFG20 [5] can reduce this complexity to $2n$ scalar multiplications, which means the dominating complexity will become $O(n)$ rather than $O(n^2)$. The aggregation slightly increases the verifier's work but the extra cost is trivial because of the succinctness of the KZG commitment scheme. These optimizations can be applied to both our PoA and PoL. Moreover, the proof length for multiple points of the KZG commitment will also be decreased to $O(1)$ if BDFG20 is integrated, but the total proof length is still $O(n)$ because of the Σ-protocol. Other advances in other PIOP systems require future research: lookup arguments, multivariate polynomials (and corresponding commitment schemes), and folding techniques.

If blockchains like Ethereum add low-gas cost support for `bls12-381`, a topic of discussion (EIP-2537[15]), verifying proofs of solvency could move on-chain. If an exchange fails to provide a smart contract with a proof of solvency in a timely fashion, the smart contract could be called to trigger penalties or other actions.

[15] `https://eips.ethereum.org/EIPS/eip-2537`

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A Cryptographic Building Blocks

This appendix describes cryptographic building blocks we use that are from the literature and not our own contribution.

A.1 Discrete Logarithm Assumption

The discrete logarithm problem describes, given a triplet $(\mathbb{G}, p, g)$, where $\mathbb{G}$ is a cyclic group of order p generated by $g \in \mathbb{G}$, and an element $y \in \mathbb{G}$, for a given adversary $\mathcal{A}$, $\mathcal{A}$ needs to compute an x such that $y = g^x$. The discrete logarithm assumption holds for $\mathbb{G}$ if it is infeasible for $\mathcal{A}$ to find such x in polynomial time.

A.2 Pedersen Commitment

The Pedersen commitment scheme [26] enables $\mathcal{P}$ to *commit* to a value x without revealing it. Pedersen commitment provides perfect hiding and computational binding based on the discrete logarithm assumption. Additionally, Pedersen commitments are *additively homomorphic*: given two commitments $\mathbf{C}_1$ and $\mathbf{C}_2$, the summation of their secrets x_1 and x_2 is the secret of $\mathbf{C}_1 \cdot \mathbf{C}_2$.

A.3 Zero-Knowledge Proofs

Informally, a zero-knowledge proof is a cryptographic protocol allowing $\mathcal{P}$ to convince $\mathcal{V}$ that the claiming statement is true without revealing additional information, except the fact that the statement's truth. A zero-knowledge proof must satisfy the following properties:

1. **Completeness:** If the statement is true, $\mathcal{V}$ will be convinced.
2. **Soundness:** If the statement is false, the probability that $\mathcal{V}$ is convinced is negligible.
3. **Honest verifier zero knowledge (HVZK):** If there exists a p.p.tsimulator $\mathcal{S}$ that can output a transcript for all possible inputs, and the distribution of this transcript is identical to the transcript generated between $\mathcal{P}$ and $\mathcal{V}$ with the same input.

A.4 Σ-Protocol

The $\Sigma-$protocol is a three-move interactive proof system. We define the $\Sigma-$protocol similarly to [13].

Definition 1 (Σ-protocol). *Let R be a binary relation between the statement x and the witness w. Given common input x to $\mathcal{P}$ and $\mathcal{V}$, and private input (x, w) such that $(x, w) \in R$ to $\mathcal{P}$, they run the following protocol:*

1. *$\mathcal{P}$ computes a message m from (x, w) and sends m.*
2. *$\mathcal{V}$ sends a random challenge c.*
3. *$\mathcal{P}$ replies with z.*

At the end of the protocol $\mathcal{V}$ has the data (x, m, c, z). He decides to output ***acc*** *or* ***rej****; such that*

- ***Completeness****: If $\mathcal{P}$ follows the protocol to generate the message (m, c, z), $\mathcal{V}$ always accepts.*
- ***Special soundness****: If there exists a p.p.t extractor $\mathcal{E}$, given any input x and any two accepting $(m, c, z), (m, c', z')$ where $c \neq c'$, $\mathcal{E}$ can compute w where $(x, w) \in R$.*
- ***Special honest verifier zero knowledge (special HVZK)****: If there exists a p.p.t simulator $\mathcal{S}$ such that additionally given the challenge c to $\mathcal{S}$, the transcript produced by $\mathcal{S}$ has the same distribution as the transcript of a conversation between the honest $\mathcal{P}$ and $\mathcal{V}$ on the same input, even if for the case that the witness for the statement does not exist.*

A commonly well-known way to convert a $\Sigma-$protocol into non-interactive is using the Fiat-Shamir transform [17], but we still use the standard interactive $\Sigma-$protocol to demonstrate our work for comprehension.

A.5 Disjunction of Σ-Protocols (OR Proof)

The disjunction of Σ-protocols (OR proof) allows $\mathcal{P}$ to prove the claimed x is x_1 or x_2 through a Σ-protocol. More precisely, given two inputs x_1, x_2, $\mathcal{P}$ proves he knows a w such that $(x_1, w) \in R_1$ or $(x_2, w) \in R_2$, but $\mathcal{V}$ cannot learn which one $\mathcal{P}$ knows. We use the same definition as [13].

Definition 2 (OR Proof). *Let s equal 0 or 1. The OR proof is a $\Sigma-$protocol that $\mathcal{P}$ and $\mathcal{V}$ are given two public inputs x_1, x_2, and $\mathcal{P}$ is given w as private input. They run the following protocol:*

1. *$\mathcal{P}$ computes the message m_s using (x_s, w) as input.*
   $\mathcal{P}$ randomly generates c_{1-s} as the challenge for x_{1-s} and runs the simulator $\mathcal{S}(x_{1-s}, c_{1-s})$ to produce (m_{1-s}, z_{1-s}).
2. *$\mathcal{P}$ sends m_s and m_{1-s}.*
3. *$\mathcal{V}$ sends a master challenge c.*
4. *$\mathcal{P}$ computes $c_s = c \oplus c_{1-s}$ and z_s on inputs (x_s, c_s, m_s, w).*
   $\mathcal{P}$ sends $(c_s, c_{1-s}, z_s, z_{1-s})$.

At the end of the protocol $\mathcal{V}$ verifies $c = c_s \oplus c_{1-s}$ and both (m_s, c_s, z_s, x_s) and $(m_{1-s}, c_{1-s}, z_{1-s}, x_{1-s})$ are valid to output ***acc*** *or* ***rej****; such that*

- ***Completeness:*** *The case of c_{1-s} is always accepted by $\mathcal{V}$ as the definition of a simulator; on the other side, the case of c_s has no difference from the standard $\Sigma-$protocol.*
- ***Special soundness:*** *Let $\mathcal{P}$ execute the protocol twice. Two accepting transcripts*

$$(x_s, x_{1-s}, c, c_s, c_{1-s}, z_s, z_{1-s}), (x_s, x_{1-s}, c', c'_s, c'_{1-s}, z'_s, z'_{1-s}), c \neq c'$$

  are given. It is clear that for some $s = 0$ or 1, the witness w such that $(x_s, w) \in R$ can be extracted through an extractor $\mathcal{E}$ by the special soundness of Σ-protocol.

– ***Special HVZK****: Given a master challenge c, let $\mathcal{S}$ choose c_s or c_{1-s} randomly and the other will be determined. Then let the simulator run twice: $\mathcal{S}(x_s, c_s), \mathcal{S}(x_{1-s}, c_{1-s})$, to output $(m_s, z_s, m_{1-s}, z_{1-s})$. The outputs of $\mathcal{S}$ have the same distribution as those of $\mathcal{P}$.*

We use $x = x_1 \vee x_2$ to denote the value x is x_1 or x_2 in the context of Σ-protocol.

A.6 Roots of Unity

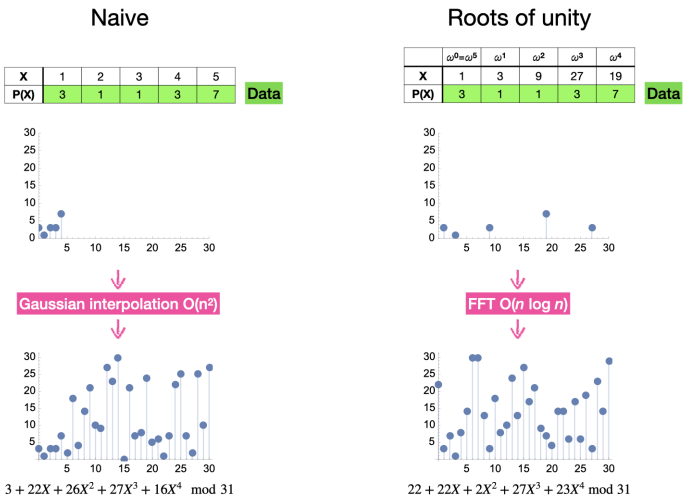

Fig. 5: Small number ($\mathbb{Z}_{31}$) example of encoding a vector of integers $\langle 3, 1, 1, 3, 7 \rangle$ into (a) the first 5 points of a polynomial, and (b) into 5th roots of unity ($\omega = 3$).

We use the approach of encoding data vectors into polynomials, committing to them using a polynomial commitment scheme (PCS), and forming zero-knowledge arguments—a model called a polynomial-based interactive oracle proof (Poly-IOP). The zk-SNARK system Plonk popularized Poly-IOPs and has many extensions and optimizations. A one-dimensional vector of data is encoded into a univariate polynomial using 1 of 3 methods (all 3 are used at different steps of Plonk): (1) into the coefficients of the polynomial, (2) as roots of the polynomial, and (3) as the y-coordinates ($\mathsf{data}_i = f(x_i)$) of points on the polynomial. Plonk mostly relies on (3) and an interpolation algorithm is used to find the corresponding coefficients of the polynomial, which is needed for the PCS. General interpolation algorithms are $O(n^2)$ work for n evaluation points but this can be reduced to $O(n \log n)$ with an optimization.

The optimization enables interpolation via the fast Fourier transform (FFT). It concerns how to choose the x-coordinates, which will serve as the index for accessing the data: evaluating $f(X)$ at x_i will reveal data_i. First note, x-coordinates are from the exponent group ($\mathbb{Z}_q$) and the choices exceed what is feasible to use (2^{255} values in `bls12-381`). Any subset can be used and interpolated. The optimization is to choose them with a mathematical structure. Specifically, instead an additive sequence (e.g., $0, 1, 2, 3, \ldots$), we use a multiplicative sequence $1, \omega, \omega \cdot \omega, \omega \cdot \omega \cdot \omega, \ldots$ or equivalently: $\omega^0, \omega^1, \omega^2, \ldots, \omega^{\kappa-1}$. Further, the sequence is closed under multiplication which means that the next index after $\omega^{\kappa-1}$ wraps back to the first index: $\omega^{k-1} \cdot \omega = \omega^{\kappa} = \omega^0 = 1$ (this property is also useful in proving relationships between data in the vector and its neighbouring values).

For terminology, we say ω is a generator with multiplicative order κ in $\mathbb{Z}_q$. This implies $\omega^{\kappa} = 1$. Rearranging, $\omega = \sqrt[\kappa]{1}$. Thus we can equivalently describe ω as a κ-th root of 1. Finally, as 1 is the unity element in $\mathbb{Z}_q$, ω is commonly called a κ-th root of unity.

For practical purposes, κ represents the length of the longest vector of data we can use in our protocol. Where does κ come from? Different elements of $\mathbb{Z}_q$ will have different multiplicative orders but every order must be a divisor of $q - 1$. Thus κ is the largest divisor of the exact value of q used in an elliptic curve standard. The value of q in `bls12-381` has $\kappa = 2^{32}$ (for terminology, this called a 2-adicity of 32).

A.7 KZG Opening with Zero-Knowledge Extension (KZG_{zk})

A polynomial commitment scheme (PCS) allows $\mathcal{P}$ to commit to a polynomial and convince $\mathcal{V}$ that the commitment is correct by testing an evaluation at a random point. Particularly, our protocol requires the scheme to use a mask polynomial to hide the polynomial, *i.e.,* the KZG commitment [21] in Pedersen form. We kindly refer to the Section 3.3 for this hiding property in the KZG paper. Although a KZG commitment does not reveal the information of the polynomial directly, the opening point leaks partial information of that polynomial. Assume a malicious verifier sends a different challenge point in each round, which allows him to recover the polynomial after $d + 1$ rounds (d is the degree of the polynomial). To explain the solution more easily, we first claim two algorithms for the KZG opening and verifying.

Claim. $\pi_\zeta \leftarrow \mathsf{KZG.Prove}(\mathcal{C}_{f_1}, \mathcal{C}_{f_2}, \ldots; f_1, f_2, \ldots; \zeta)$. This is an algorithm for $\mathcal{P}$ that takes as input polynomials $\{f_1, f_2, \ldots\}$, the corresponding commitments $\{\mathcal{C}_{f_1}, \mathcal{C}_{f_2}, \ldots\}$, and an evaluation point ζ to output a proof π_ζ to prove the opening evaluations are correct. To do so, $\mathcal{P}$ simply opens each polynomial at ζ using the KZG opening algorithm.

Claim. $\{1/0\} \leftarrow \mathsf{KZG.Verify}(\mathcal{C}_{f_1}, \mathcal{C}_{f_2}, \ldots; \pi_\zeta; \zeta)$. This is an algorithm for $\mathcal{V}$ that takes as input the commitments to $f_1, f_2, \ldots$, π_ζ and the evaluation point ζ to verify π_ζ. If π_ζ is valid, it returns 1, else it returns 0. In other words, $\mathcal{V}$ verifies the opening proof using the KZG verifying algorithm.

Now we define KZG_{zk}.

Definition 3 (KZG_{zk})**.** KZG_{zk} *is a variant of KZG commitment scheme such that* $\mathcal{P}$ *is given input* $f \in \mathbb{F}_{<d}[X]$*,* $\mathcal{P}$ *wants to open* f *at* n *random points, where* $n \leq d+1$*. They run the protocol as follows:*

1. $\mathcal{P}$ *generates* n *random numbers* $x_1, x_2, \ldots, x_n \in \mathbb{F} \setminus H$ *(*H *is the domain of* f*) and another* n *random numbers* $y_1, y_2, \ldots, y_n \in \mathbb{F}$*.*
2. $\mathcal{P}$ *incrementally interpolates* f *at* n *more points* $x_1, x_2, \ldots, x_n$ *such that*

$$f(x_1) = y_1, f(x_2) = y_2, \ldots, f(x_n) = y_n$$

3. $\mathcal{P}$ *publishes the commitment* $\mathcal{C}_f$*.*
4. $\mathcal{V}$ *sends* n *random evaluation points* $\zeta_1, \zeta_2, \ldots, \zeta_n \in \mathbb{F} \setminus H$*.*
5. *For each* $\zeta_i, i \in [1, n]$*,* $\mathcal{P}$ *computes* $\pi_{\zeta_i} \leftarrow \mathsf{KZG.Prove}(f; \zeta_i)$ *to open* f *at* ζ_i*.*
6. $\mathcal{V}$ *outputs* ***acc*** *if and only if* $\mathsf{KZG.Verify}(f; \pi_{\zeta_i}; \zeta)$ *returns 1 for each* $\pi_{\zeta_i}, i \in [1, n]$*.*

When we say polynomials $\{f_1, f_2, \ldots\}$ are committed using KZG_{zk}, we mean $\mathcal{P}$ and $\mathcal{V}$ will run the above protocol to prove the opening evaluations of $\{f_1, f_2, \ldots\}$ are correct.

Theorem 2. KZG_{zk} *is complete, sound, and HVZK.*

Proof. Completeness is clear because the new polynomial has the same evaluations as the old one over the domain.

For soundness, note the difference between the variant and the original is we reduce the field of the challenge evaluation point from $\mathbb{F}$ to $\mathbb{F} \setminus H$. The soundness error increases but is still negligible. We argue it is sound rather than knowledge sound as sampling the challenge point outside H does not require the prover to know the original polynomial.

To verify zero knowledge, we construct a simulator $\mathcal{S}$. Let $\mathcal{S}$ construct a vanishing polynomial f^* over the same domain and randomly generate $\{x_1^*, x_2^*, \ldots, x_n^*\}$, $\{y_1^*, y_2^*, \ldots, y_n^*\}$ like $\mathcal{P}$, and then incrementally interpolate f^* such that

$$f^*(x_1^*) = y_1^*, f^*(x_2^*) = y_2^*, \ldots, f^*(x_n^*) = y_n^*$$

We can observe when $\mathcal{V}$ interacts with $\mathcal{S}$ to execute the protocol, $\mathcal{V}$ always accepts the proof from $\mathcal{S}$ because f^* has the same roots as f. Given $\{x_1^*, x_2^*, \ldots, x_n^*\}$, $\{y_1^*, y_2^*, \ldots, y_n^*\}$ are chosen uniformly at random each time, that is exactly the same as $\mathcal{P}$ incrementally interpolates f. Thus $\mathcal{V}$ cannot distinguish between the transcript from $\mathcal{S}$ and the transcript from $\mathcal{P}$.

A.8 Range Proof for Single Value

Boneh *et al.* [6] introduced how to prove a number x is in the range $[0, 2^k)$ without revealing x. Note this range proof is for proving a single value, but it is natural to extend this idea for multiple values as our work did. Here we do not describe how to use it for multiple values to avoid redundancy since it is introduced in Protocol 4. We recap the rangeproof from Boneh *et al.* based on `PIOP`:

1. $\mathcal{P}$ is given a value x and $\mathcal{V}$ is given the commitment to $g(X)$ where $g(\omega^0) = x$ (ω is the root of unity over an FFT field H).
2. $\mathcal{P}$ decomposes x to a vector of binary digits $\overline{z} = \langle z_1, z_2, \ldots, z_k \rangle$, so that $x = \sum_{i=0}^{k-1} 2^i \cdot z_i$.
3. $\mathcal{P}$ constructs a vector $\overline{x} = \langle x_1, x_2, \ldots, x_k \rangle$ such that

$$\begin{aligned} x_1 &= x \\ x_k &= z_k \\ x_i &= 2x_{i+1} + z_i, i \in [1, k-1] \end{aligned}$$

4. $\mathcal{P}$ interpolates a polynomial f from $\overline{x}$.
5. $\mathcal{P}$ proves the following relations hold using PIOP

$$\begin{aligned} \mathcal{R}_1 &= \left\{ f, g \;\middle|\; [f(X) - g(X)] \cdot \frac{X^n - 1}{X - \omega^0} = 0 \right\} \\ \mathcal{R}_2 &= \left\{ f \;\middle|\; f(X) \cdot [f(X) - 1] \cdot \frac{X^n - 1}{X - \omega^{n-1}} = 0 \right\} \\ \mathcal{R}_3 &= \left\{ f \;\middle|\; [f(X) - 2 \cdot f(X\omega)] \cdot [f(X) - 2 \cdot f(X\omega) - 1] \cdot (X - \omega^{n-1}) = 0 \right\} \end{aligned}$$

Lemma 1. *The above range proof is complete, has knowledge soundness in the algebraic group model and HVZK.*

Proof. Completeness is clear by following the protocol. Knowledge soundness and HVZK are direct results from the definition of PIOP and KZG commitment scheme.

It is worth noting that Boneh *et al.* [6] used a polynomial commitment to hide x. However, we realized that it is not necessary because we can use a Pedersen commitment to hide x and prove the commitment is correct using the KZG commitment scheme. We refer to Section A.9 for more details.

A.9 Open KZG with Committed Value (KZG_{cm})

KZG_{zk} allows $\mathcal{P}$ to prove a polynomial is vanishing over a specified domain. However, in some cases, $\mathcal{P}$ needs to prove the claimed evaluation is correct. For example, we construct a polynomial where the evaluation at ω^0 is the total assets we want to prove. Instead of revealing the evaluation directly, we publish the committed value. Now we describe this opening scheme following the idea in [2].

Definition 4 (KZG_{cm})**.** KZG_{cm} *is a KZG opening scheme such that* $\mathcal{P}$ *takes as input* $f \in \mathbb{F}_{<d}[X]$*.* $\mathcal{P}$ *and* $\mathcal{V}$ *run the protocol as follows:*

1. $\mathcal{P}$ *generates a random polynomial* $\hat{f}$ *with the same degree as* f *and computes the commitment to* f*,* $\mathcal{C}_f = g_1^{f(\tau)} h_1^{\hat{f}(\tau)}$*.*
2. $\mathcal{V}$ *sends a random evaluation point* a *as challenge.*

3. $\mathcal{P}$ *computes the witness* w *for* a *such that*

$$w = g_1^{\psi(\tau)} h_1^{\hat{\psi}(\tau)}$$

where $\psi(x) = \frac{f(X)-f(a)}{X-a}$, $\hat{\psi}(x) = \frac{\hat{f}(X)-\hat{f}(a)}{X-a}$.

4. $\mathcal{P}$ *sends* w *and* $\boldsymbol{C}(b)$ *such that* $\boldsymbol{C}(b) = g_1^{f(a)} h_1^{\hat{f}(a)}$.
5. $\mathcal{V}$ *outputs* ***acc*** *if and only if*

$$e(\mathcal{C}/\boldsymbol{C}(b), [1]_2) = e(w, [\tau - a]_2)$$

Theorem 3. KZG_{cm} *is complete, knowledge sound, and HVZK.*

Proof. Completeness follows the original KZG commitment scheme.

For knowledge soundness, KZG_{cm} does not violate the knowledge soundness of the original KZG commitment scheme. Recall the computational binding property of Pedersen commitment, which means it is infeasible for $\mathcal{P}$ to compute a b^* such that $f(a) \neq b^*, \mathbf{C}(b) = \mathbf{C}(b^*)$ based on discrete logarithm assumption.

To prove HVZK, let the simulator $\mathcal{S}$ compute

$$\mathcal{C}_{f^*} \xleftarrow{\$} \mathbb{G}_1, a^* \xleftarrow{\$} \mathbb{F}, \mathbf{C}(b^*) \xleftarrow{\$} \mathbb{G}_1, w^* = \mathcal{C}_{f^*}/(\mathbf{C}(b^*) \cdot [\tau - a^*]_1)$$

It is clear that the simulated conversation $(\mathcal{C}_{f^*}, a^*, \mathbf{C}(b^*), w^*)$ is always accepted by $\mathcal{V}$. We can observe that $\mathcal{C}_{f^*}, a^*, \mathbf{C}(b^*)$ are independent, uniformly distributed over their own field, and w^* is determined by $\mathcal{C}_{f^*}/(\mathbf{C}(b^*) \cdot [\tau - a^*]_1)$. Thus, the simulated conversation has the same distribution as the output of $\mathcal{P}$.

We claim two algorithms representing the step 4 and 5 in the above opening scheme, respectively.

Claim. $\mathbf{C}(b) \leftarrow \mathsf{KZG}_{cm}.\mathsf{Prove}(f; b; a)$. This is an algorithm for $\mathcal{P}$ that takes as input a polynomial f and an evaluation point a to output the committed evaluation of $b = f(a)$.

Claim. $\{1/0\} \leftarrow \mathsf{KZG}_{cm}.\mathsf{Verify}(\mathcal{C}_f; \mathbf{C}(b); a)$. This is an algorithm for $\mathcal{V}$ that takes as input the commitment to f, the committed evaluation $f(a)$, and the point a to verify the committed value. If $\mathbf{C}(b)$ is the correct evaluation at a, it returns 1, else it returns 0.

A.10 Polynomial Relation

Definition 5 (Polynomial Relation). *A polynomial relation* $\mathcal{R}$ *is a set of tuples* $(f_1, f_2, \ldots, f_t)$ *such that* $f_1, f_2, \ldots, f_t \in \mathbb{F}_{<d}[X]$ *and the tuple satisfies a certain equation.*

A.11 Polynomial Protocol

Definition 6 (Polynomial Protocol). *Fix positive integer d, t, l. Let $i \in [1, l]$. Let $\mathcal{R} \subseteq \mathbb{F} \times \mathbb{F} \times \cdots \times \mathbb{F}$ be a polynomial relation for one or more polynomials. Given a set of polynomials $f_1, f_2, \ldots, f_t \in \mathbb{F}_{<d}[X]$ as $\mathcal{P}$'s private input, and a set of polynomial relations $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_l$ as public input, a polynomial protocol is a three-move protocol that $\mathcal{P}$ wants to convince $\mathcal{V}$ each $\mathcal{R}_i$ holds for the certain set $F_i \subseteq \{f_1, f_2, \ldots, f_t\}$. $\mathcal{P}$ and $\mathcal{V}$ runs the protocol as follows:*

- *$Setup(d, t, l) \rightarrow (\mathtt{pk}, \mathtt{vk})$*
  - *Given the maximum degree d of each polynomial, the number of polynomials t, and the number of relations l, outputs a proving key $\mathtt{pk}$ and a verification key $\mathtt{vk}$ for the polynomial protocol.*
- *$Prove(\mathtt{pk}, \mathcal{R}_i, F_i) \rightarrow \pi_i$*
  1. *$\mathcal{P}$ commits to $f_1, f_2, \ldots, f_t$ with $\mathtt{pk}$ using KZG_{zk}, and publishes all commitments, $\mathcal{C}_{f_1}, \mathcal{C}_{f_2}, \ldots, \mathcal{C}_{f_t}$.*
  2. *$\mathcal{V}$ sends a random evaluation point as the challenge.*
  3. *$\mathcal{P}$ responds with the corresponding evaluation and the commitment to the witness at the evaluation point for each polynomial.*
- *$Verify(\mathtt{vk}, \mathcal{R}_i, \mathcal{C}_{F_i}, \pi_i) \rightarrow b$*
  - *$\mathcal{V}$ outputs $b = \textbf{\textit{acc}}|\textbf{\textit{rej}}$ by checking*
    1. *Each evaluation of $f_1, f_2, \ldots, f_t$ at the random point is valid through the KZG verifying algorithm.*
    2. *Each relation $\mathcal{R}_i$ holds for the prescribed set F_i.*

A polynomial protocol has the following properties:

- ***Completeness:** $\mathcal{V}$ always outputs **acc** if $\mathcal{P}$ follows the protocol correctly to compute the proof π_i for the relation $\mathcal{R}_i$, and $\mathcal{R}_i$ holds for the prescribed polynomials F_i, denoted by $(F_i, \pi_i) \in \mathcal{R}_i$.*
- ***Knowledge soundness in the algebraic group model**: For any algebraic adversary $\mathcal{A}$ in a polynomial protocol, there exists a p.p.t extractor $\mathcal{E}$ given access to $\mathcal{A}$'s messages during the protocol, and $\mathcal{A}$ can win the following game with negligible probability:*
  1. *Given the inputs that $\mathcal{P}$ can access, $\mathcal{A}$ outputs $\mathcal{C}_{f_1}, \mathcal{C}_{f_2}, \ldots, \mathcal{C}_{f_t}$.*
  2. *$\mathcal{E}$ outputs $f_1, f_2, \ldots, f_t \in \mathbb{F}_{<d}[X]$ from $\mathcal{A}$'s output and $\{F_i\}$ from these polynomials.*
  3. *$\mathcal{A}$ outputs the evaluation at the random evaluation point for each polynomial and the corresponding proofs $\{\pi_i\}$.*
  4. *$\mathcal{A}$ wins if*
     - *$\mathcal{V}$ accepts the proof at the end of the protocol.*
     - *$(F_i, \pi_i) \notin \mathcal{R}_i$ or any evaluation is not correct.*
- ***HVZK**: There exists a p.p.t simulator $\mathcal{S}$ such that for any p.p.t adversary $\mathcal{A}$, the output of $\mathcal{A}$ when interacting with $\mathcal{P}$ is computationally indistinguishable from the output of $\mathcal{A}$ when interacting with $\mathcal{S}$.*

Note that there are some batched KZG opening schemes [19,5,18] to prove several relations more efficiently. Our implementation leverages the idea from [19] but we do not discuss them in the paper as it is not the purpose of this paper.

B Performance

B.1 Theoretical Performance

In this section, we analyze the performance of Xiezhi, and compare the theoretical performance of our work with other prior schemes as actual runtimes vary depending on specific implementations and optimizations. Our analysis ignores the relatively trivial cost like Fast Fourier Transform (FFT) interpolation and focuses on the heavy work such as scalar multiplication and group operations. Also, we assumes each protocol is executed in a single thread.

Proof of Assets We use κ to denote the size of the anonymity set and we assume κ is the power of two for simplicity. The performance analysis of π_{keys} and π_{users} are performed as follows:

- π_{keys}: When opening an evaluation of a KZG commitment for each public key, one multi-scalar multiplication (MSM) for the witness and one MSM for the blinding polynomial are involved. The number of scalar multiplications of the Σ-protocol is constant. Thus, the overhead proving time of π_{keys} is $O(\kappa^2)$. In terms of the verifier's work, for each key, $\mathcal{V}$ performs scalar multiplications for constant times and manipulates the batched KZG scheme to validate related polynomial constraints, which means $O(1)$ verifying time and proof size. Therefore, the overhead verifying time and proof size of π_{keys} is $O(\kappa)$.
- π_{assets}: $\mathcal{P}$ constructs the accumulator and commits to a constant number of polynomials. Since $\mathcal{P}$ opens one point of each polynomial, the proving time is $O(\kappa)$ and the proof size is $O(1)$. While it takes constant time for $\mathcal{V}$ to verify the proof of the PCS, $\mathcal{V}$ needs to interpolate the balances and commit to the balance polynomial, which means one MSM is involved. Therefore, the overhead verifier's work of π_{assets} is $O(\kappa)$.

Proof of Liability We use μ to denote the number of users and k to denote the allowed size of the range proof. When $\mathcal{P}$ computes the accumulative polynomial to prove the total liablity is correct, it can be done in linear time. Different from π_{keys}, $\mathcal{P}$ only opens each polynomial at one random evaluation point. Thus, the proving time is $O(\mu)$.

The verifier's work is broken into π_{users} and $\pi_{\mathsf{liabilities}}$:

- π_{users}: Each user verifies his balance is the evaluation of the polynomial p_1 and his user identifier is the evaluation of the polynomial f_{uid}. The user checks two KZG proofs, so the proof size and the verifying time for customers are both $O(1)$.
- $\pi_{\mathsf{liabilities}}$: Auditor verifies the constraints among polynomials $\{p_i\}$ are correct and the committed total liabilities is the evaluation of $f_{\mathsf{liab}}(\omega^0)$. The first step can be done in $O(k)$ as the number of polynomials is related to the range proof rather than the number of users. The second step involves opening one KZG commitment through KZG_{cm}, which means the verifying time and the proof size for auditors are both $O(k)$.

π_{keys}						
Scheme	**Proving time**	**Verifying time**	**Proof size**	**Hashed**	**NI**	**Full PoS**
Xiezhi (**Ours**)	$O(\kappa^2)$	$O(\kappa)$	$O(\kappa)$	✓	✓	✓

π_{assets}						
Scheme	**Proving time**	**Verifying time**	**Proof size**	**Hashed**	**NI**	**Full PoS**
Provisions[12]	$O(\kappa)$	$O(\kappa)$	$O(\kappa)$	✗	✓	✓
Bulletproofs[7]	$O(\kappa)$	$O(\kappa)$	$O(\log \kappa)$	✓	✓	✓
CompositeNIZK[1]	$O(\kappa \log \kappa)$	$O(\kappa)$	$O(\kappa)$	✓	✓	✓
gOTzilla[3]	$O(\kappa)$	$O(\kappa)$	$O(\log \kappa)$	✓	✗	✗
IZPR[11]	$O(t \log t)$	$O(1)$	$O(1)$	✓	✓	✗
Xiezhi (**Ours**)	$O(\kappa)$	$O(1)$	$O(1)$	✓	✓	✓

Table 2: Comparison of this work with prior PoA schemes. π_{keys} only appears in Xiezhi and IZPR (IZPR calls it bootstrapping), but IZPR did not specify how the bootstrapping works. **Hashed**: whether the scheme is compatible with hashed keys. **NI**: non-interactive. **Full PoS**: whether the scheme also has a protocol for proof of liabilities (if not, the row is greyed out to indicate it is a point of reference but does not compete directly with Xiezhi). Notation: κ is the size of the anonymity set that the exchange wants to prove. For IZPR[11], t is the throughput of the blockchain (number of addresses which have changed since the last proof).

Comparison In Table 2, we compare this work with other prior PoA schemes. Both IZPR and this work utilize bootstrapping, but the bootstrapping of IZPR will be introduced in their following paper. We only analyze the performance of the bootstrapping for this work. In Table 3, we compare this work with prior PoL schemes.

C Security Proofs for Xiezhi's Components

C.1 Definitions

Definitions 7 and 8 are taken largely verbatim from the Provisions paper at CCS 2015 [12]. Let $\mathcal{A}$ (exchange-controlled addresses) and $\mathcal{A}'$ (anonymity set of addresses) denote mappings $(y = g^x) \mapsto \mathrm{bal}(y)$ where $\mathcal{A} \subseteq \mathcal{A}'$, y is the public key corresponding to an Ethereum address with private key x and $\mathrm{bal}(y)$ is the amount of currency, or assets, observably spendable by this key on the blockchain. Let $\mathcal{L}$ denote a mapping $\mathfrak{uid} \mapsto \ell$ where ℓ is the amount of currency, or liabilities, owed by the exchange to each user identified by the unique identity $\mathfrak{uid}$. A balance is a positive integer in $[0, \mathsf{MaxETH}]$ for a known upper-bound MaxETH. The size of $\mathcal{A}'$ is known, the size of $\mathcal{A}$ is generally unknown (beyond being less than or equal to $\mathcal{A}'$), and the size of $\mathcal{L}$ is generally unknown (see Definition 8(4) below).

Definition 7 (Valid Pair). *We say that $\mathcal{A}$ and $\mathcal{L}$ are a valid pair with respect to a positive integer* MaxETH *iff* $\forall \mathfrak{uid} \in \mathcal{L}$,

Scheme	Proving time	Verifying time		Proof size		Full PoS
		π_{users}	$\pi_{\mathsf{liabilities}}$	π_{users}	$\pi_{\mathsf{liabilities}}$	
Provisions[12]	$O(\mu)$	$O(1)$	$O(\mu)$	$O(1)$	$O(\mu)$	✓
Bulletproofs[7]	$O(\mu)$	$O(1)$	$O(\mu)$	$O(1)$	$O(\log \mu)$	✓
CompositeNIZK[1]	$O(\mu)$	$O(1)$	$O(\mu)$	$O(1)$	$O(\mu)$	✓
DAPOL+[20]	$O(\mu \log \mu)$	$O(\log \mu)$	$O(1)$	$O(\log \mu)$	$O(1)$	✗
SSVT-based[16]	$O(\log_\lambda \mu)$	$O(\log_\lambda \mu)$	$O(1)$	$O(\log_\lambda \mu)$	$O(1)$	✗
Notus[27]	$O(\mu \log \mu)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	✗
Xiezhi (**Ours**)	$O(k \cdot \mu)$	$O(1)$	$O(k)$	$O(1)$	$O(k)$	✓

Table 3: Comparison of this work with prior PoL schemes. **Full PoS**: whether the scheme has existing protocol for proof of assets (if not, the row is greyed out to indicate it is a point of reference but does not compete directly with Xiezhi). Notation: μ is the number of users, k is the bit length of the maximum account balance (which we keep visible but note is effectively a constant). For SSVT-based [16], λ is the arity of the Verkle Tree it uses.

- $\sum_{y \in \mathcal{A}} \mathcal{A}[y] - \sum_{\mathfrak{uid} \in \mathcal{L}} \mathcal{L}[\mathfrak{uid}] \geq 0$
- $0 \leq \mathcal{L}[\mathfrak{uid}] \leq \mathsf{MaxETH}$

Consider an interactive protocol ProveSolvency run between an exchange $\mathcal{E}$ and user $\mathcal{U}$ such that

- $\mathsf{output}_{\mathcal{E}}^{\mathsf{ProveSolvency}}(1^k, \mathsf{MaxETH}, \mathcal{A}, \mathcal{L}, \mathcal{A}') = \emptyset$
- $\mathsf{output}_{\mathcal{U}}^{\mathsf{ProveSolvency}}(1^k, \mathsf{MaxETH}, \mathcal{A}', \mathfrak{uid}, \ell) \in \{ACCEPT, REJECT\}$

For brevity, we refer to these as $\mathrm{out}_{\mathcal{E}}$ and $\mathrm{out}_{\mathcal{U}}$ respectively. Next we define, with reference to the valid pair definition, a privacy-preserving proof of solvency.

Definition 8 (Privacy-Preserving Proof of Solvency). *A privacy-preserving proof of solvency is a probabilistic polynomial-time interactive protocol ProveSolvency, with inputs/outputs as above, such that the following properties hold:*

1. *Correctness. If $\mathcal{A}$ and $\mathcal{L}$ are a valid pair and $\mathcal{L}[\mathfrak{uid}] = \ell$, then $\Pr[out_{\mathcal{U}} = ACCEPT] = 1$.*
2. *k-Soundness. If $\mathcal{A}$ and $\mathcal{L}$ are instead not a valid pair, or if $\mathcal{L}[\mathfrak{uid}] \neq \ell$, then $\Pr[out_{\mathcal{U}} = REJECT] \geq 1 - \mathsf{negl}(k)$.*
3. *Ownership. For all valid pairs $\mathcal{A}$ and $\mathcal{L}$, if $\Pr[out_{\mathcal{U}} = ACCEPT] = 1$, then the exchange must have 'known' the private keys associated with the public keys in $\mathcal{A}$; i.e., there exists an extractor that, given $\mathcal{A}$, $\mathcal{L}$, and rewindable black-box access to exchange $\mathcal{E}$, can produce x for all $y \in \mathcal{A}$.*
4. *Privacy. A potentially dishonest user $\mathcal{U}'$ interacting with an honest exchange $\mathcal{E}$ cannot learn anything about a valid pair $\mathcal{A}$ and $\mathcal{L}$ beyond its validity and $\mathcal{L}[\mathfrak{uid}]$ (and possibly $|\mathcal{A}|$ and $|\mathcal{L}|$); i.e., even a cheating user cannot distinguish between an interaction using the real pair $\mathcal{A}$ and $\mathcal{L}$ and any other (equally sized) valid pair $\hat{\mathcal{A}}$ and $\hat{\mathcal{L}}$ such that $\hat{\mathcal{L}}[\mathfrak{uid}] = \mathcal{L}[\mathfrak{uid}]$.*

C.2 Theorems

Theorem 4. *A Σ-protocol for relation* ZKPoK$\{(\mathsf{sk}_i, s_i) : [\mathsf{pk}_i = g^{\mathsf{sk}_i} \wedge \boldsymbol{C}_{bls}(s_i) = \boldsymbol{C}_{bls}(1)] \vee \boldsymbol{C}_{bls}(s_i) = \boldsymbol{C}_{bls}(0)\}$ *exists which is complete, has special soundness and is special HVZK.*

Proof. To demonstrate completeness, consult Protocol 1.

To demonstrate special soundness, let two accepting conversations between $\mathcal{P}$ and $\mathcal{V}$

$$(t_1, t_2, t_3, e, e_0, e_1, z_1, z_2, z_3), (t_1, t_2, t_3, e', e'_0, e'_1, z'_1, z'_2, z'_3) \text{ with } e \neq e'$$

be given. It is obvious for some $s = 0$ or 1 and $e_s \neq e'_s$, we can compute sk_i, s_i from the above conversations. Thus, the Σ-protocol for the relation ZKPoK has special soundness.

To demonstrate special HVZK, given e and randomly choose e_0, e_1 such that $e = e_0 \oplus e_1$, let the simulator compute

$$z_1 \xleftarrow{\$} \mathbb{Z}_{\mathtt{secp}}, z_2 \xleftarrow{\$} \mathbb{Z}_{\mathtt{bls}}, z_3 \xleftarrow{\$} \mathbb{Z}_{\mathtt{bls}},$$
$$t_1 = g_{\mathtt{secp}}^{z_1} / \mathsf{pk}_i^{e_0}, t_2 = g_{\mathtt{bls}}^{e_0} h_{\mathtt{bls}}^{z_2} / p_i^{e_0}, t_3 = h_{\mathtt{bls}}^{z_3} / p_i^{e_1}$$

and output $(t_1, t_2, t_3, e, e_0, e_1, z_1, z_2, z_3)$. Clearly, the transcript is accepted by $\mathcal{V}$. Note that e, e_0, and e_1 are random t-bit strings, which means they have the same distribution as the conversation between $\mathcal{P}$ and $\mathcal{V}$. z_1, z_2, and z_3 are uniformly distributed over their corresponding fields; moreover, given $(e_0, e_1, z_1, z_2, z_3)$, (t_1, t_2, t_3) are uniquely determined by the above equations. Therefore, the simulated transcript is not distinguishable from the real one to $\mathcal{V}$.

Corollary 1. *A Σ-protocol for relation* ZKPoK$\{(\mathsf{sk}_i, s_i) : [\mathsf{pk}_i = g^{\mathsf{sk}_i} \wedge \boldsymbol{C}_{bls}(s_i) = \boldsymbol{C}_{bls}(1)] \vee \boldsymbol{C}_{bls}(s_i) = \boldsymbol{C}_{bls}(0)\}$ *exists which is a non-interactive zero knowledge proof (NIZKP).*

Proof. Given the relation can proven with a "standard" Σ-protocol (Theorem 4), we can use the well-known Fiat-Shamir heuristic to compile it to a NIZKP in the random oracle model. We do not repeat the proof for this (see [13,24]) but stress that strong Fiat-Shamir [4] needs to be used here and in the Poly-IOP components of Xiezhi, or practical attacks could be leveraged against the system (*cf.* [14]).

Theorem 5. *A polynomial protocol with the zero-knowledge extension KZG_{zk} is complete, has knowledge soundness in the algebraic group model, and is HVZK.*

Proof. Completeness is clear: for an honest $\mathcal{P}$, the evaluations of polynomials are correct and the relations also hold. Thus, $\mathcal{V}$ will always accept the proofs.

We argue the knowledge soundness from two aspects: the evaluations and the relations. The binding property of KZG commitment tells us the probability that any invalid evaluation passes the verifying is negligible, which means $\mathcal{A}$ can win the first condition of the attack game with extremely low probability. By the

Schwartz-Zippel lemma, the equation defined by a relation $\mathcal{R}$ has overwhelmingly low probability to hold if the evaluation at a random point does not satisfy the equation. Therefore, the knowledge soundness is proved.

Since $\mathcal{V}$ only knows the commitments to the polynomials and the witnesses and the opening evaluations, the commitments leak no information of the polynomials and the witnesses because of the hiding property of KZG commitment. By Theorem 2, the opening scheme is HVZK. Thus, the polynomial protocol with KZG_{zk} is HVZK.

C.3 Corollaries

Corollary 2. π_{keys} *is complete, sound, and HVZK.*

Proof. Recall that the π_{keys} argument contains the relation proven to be complete, sound, and HVZK in Theorem 4. It remains to be shown the rest of the protocol (Protocol 2) is secure.

Completeness follows from Protocol 2. The remainder of the protocol involves $\mathcal{P}$ demonstrating that the selector polynomial encodes a 1 at index ω^i if and only if the corresponding i-th run of the Σ-protocol used $s = 1$, and contains a 0 otherwise.

For π_{keys} to be sound, it requires (i) the polynomial commitment scheme (PCS) to be binding and (ii) the PCS to have a sound point-evaluation argument. These two properties are both demonstrated for KZG in the original paper [21]. Specifically these two properties rely on four assumptions:

- **KZG.A1**: The trusted setup outputs a structured reference string **srs** with the value of τ unknown to $\mathcal{P}$.
- **KZG.A2**: The value of τ cannot be extracted from **srs** which assumes $\mathcal{P}$ is computationally bounded and relies on (for us in `bls12-381`) the t-strong Diffie-Hellman (t-SDH) assumption.
- **KZG.A3**: If an adversary interpolates a polynomial through the point (ω^i, y) such that $y = f(\omega^i)$ but claims $y' = f(\omega^i)$ for some $y' \neq y$ then the probability that $\tau - y'$ evenly divides $y = f(\tau)$ is overwhelmingly low. This property can be demonstrated using the Schwartz-Zippel lemma by showing the number of τ values satisfying this property is bounded from above by d/q where d is the degree of the polynomial and q is the size of the exponent group. For `bls12-381` with 255-bit exponents and 2-adicity of 32, this is close to $2^{32-255} = 2^{-233}$ which is negligible.

Finally, in our protocol $\mathcal{P}$ does not reveal the evaluation of the polynomial at a point, $\mathcal{P}$ instead reveals a commitment to the evaluation through KZG_{cm}. In the original KZG opening scheme, $\mathcal{P}$ opens the commitment to the polynomial first and then opens the evaluation at the challenge point. Our modification just moves the computation work for the committed evaluation from $\mathcal{V}$ to $\mathcal{P}$. By Theorem 3, KZG_{cm} is HVZK. Therefore, π_{keys} has zero knowledge.

For future claims, we encapsulate all assumptions about KZG as a *polynomial oracle*.

Corollary 3. *π_{assets} is complete, has knowledge soundness in the algebraic group model, and is HVZK.*

Proof. Clearly, π_{assets} is a polynomial protocol for two polynomial relations (i) $f_{\mathsf{assets}}(X) - f_{\mathsf{assets}}(X\omega) = f_{\mathsf{bal}}(X) \cdot f_{\mathsf{sel}}(X), X \neq \omega^{n-1}$ and (ii) $f_{\mathsf{assets}}(\omega^{n-1}) = f_{\mathsf{bal}}(\omega^{n-1}) \cdot f_{\mathsf{sel}}(\omega^{n-1})$. The first relation proves the starting values are the same, and the second proves each successive value in the accumulative vector adds its adjacent value with the corresponding value. π_{assets} leverages `PIOP` to prove the relations are correct. Additionally, to complete the PoA proof, π_{assets} publishes the evaluation of $f_{\mathsf{assets}}(\omega^0)$ through KZG_{cm}. We already analyzed the security of KZG_{cm} in Theorem 3. Thus, π_{assets} is complete, knowledge sound, and HVZK by Definition 6 and Theorem 5.

Corollary 4. *$\pi_{\mathsf{liabilities}}$ is complete, has knowledge soundness in the algebraic group model, and is HVZK.*

Proof. Completeness follows from Protocol 5.

Given a *polynomial oracle*, $\mathcal{P}$ commits to a set of integers in binary form and builds a vector to accumulate the bits into the integer representation (call this the range accumulator). Knowledge soundness of this aspect follows from the knowledge soundness of the range proof by Lemma 1 which uses the *polynomial oracle* to demonstrate three constraints: that the range accumulator starts with a 0 or 1; that the binary relationship between adjacent bits in the range accumulator are 0 or 1; and that the header of the range accumulator matches a standalone commitment to the integer (we do not use this, we just use the header values directly from $p_1(X)$). To complete soundness, $\mathcal{V}$ must check that no more than k bits are used for an integer in $[0, k)$. Outside of the range proof, $\mathcal{P}$ builds a vector ($f_{\mathsf{liab}}(X)$) to accumulate the sum of each header value ($p_1(X)$) from the set of range accumulators for each user account. This is the same protocol as in π_{assets}.

For HVZK, it is similar to the proof of π_{assets}.

Corollary 5. *π_{users} is complete, has knowledge soundness in the algebraic group model, and is HVZK.*

Proof. Completeness follows from Protocol 6.

Knowledge soundness follows directly from the *polynomial oracle* which, for π_{users}, opens two points at the same index on two polynomials—one demonstrates the user's balance and one demonstrates the user's identification. The sufficiency of this to bind the balance to the user ID is already proven in Provisions which uses the same mechanism (for a different commitment scheme). The KZG assumptions already addressed in Corollary 2 cover the rest.

To verify HVZK, recall the properties of KZG commitments—seeing a polynomial commitment and an opening at a specific evaluation point reveals no further information about any other point on the polynomial. KZG does not reveal the degree of the polynomial, which would provide the number of users of the exchange, but an upperbound exists in the size of **srs** from the trusted

setup (and if it can be assumed the prover will act efficiently, the largest root of unity). For each user, a p.p.t simulator can be constructed such that the evaluation at the user's index is equal to the user's balance/identification while other evaluations are random numbers. As a user (the verifier), he cannot distinguish between the simulated transcript and the real one due to the hiding property of KZG commitment.

Corollary 6. π_{solvency} *is complete, sound, and HVZK.*

Proof. Completeness follows from Protocol 7. The soundness of the argument is that $f_{\mathsf{assets}}(\omega^0)$ is sound under Corollary 3, $f_{\mathsf{liab}}(\omega^0)$ is sound under Corollary 4, and $f_{\mathsf{eq}}(\omega^0)$ is zero or positive by the soundness of the range proof (as addressed in Corollary 4). The overall constraint demonstrates that the total assets equal or exceed the total liabilities. HVZK similarly follows from the same previous corollaries (3, 4, and range proof).

C.4 Proof of Theorem 1

Proof. To prove this theorem, we rely on the corollaries in Section C.3. There are no new insights, it is simply a matter of mapping what is proven in the corollaries onto what is required in the definition of a privacy-preserving proof of solvency.

1. *Correctness.* If π_{solvency} is complete (Corollary 6), then Xiezhi is correct according to Definition 8.
2. *k-Soundness.* If $\mathcal{A}$ and $\mathcal{L}$ are not a valid pair and the protocol accepts with probability greater than $\mathsf{neg}(k)$, then π_{solvency} is not sound (contradicting Corollary 6), where soundness is bounded by $k = \min[d/n, 2^{-t}]$ where $d/n = 2^{-233}$ (Schwartz-Zippel lemma for polynomial commitments in `bls12-381`) and $t = 2^{-254}$ (challenge length for NIZKPs under Fiat-Shamir for a common challenge in `secp256k1` and `bls12-381`). If $\mathcal{L}[\mathfrak{uid}] \neq \ell$ (*i.e.,* the exchange provides the user with the wrong balance) and the protocol accepts with probability greater than $\mathsf{neg}(k)$, then π_{solvency} is not sound (contradicting Corollary 6).
3. *Ownership.* Recall that ownership means that if the protocol accepts, there exists an extractor that can produce x for all $y \in \mathcal{A}$. We show such an extractor in the proof of Theorem 4.
4. *Privacy.* Roughly, this means a (statically) corrupted user cannot distinguish between an interaction using the real pair $\mathcal{A}$ and $\mathcal{L}$ and any other (equally sized) valid pair $\hat{\mathcal{A}}$ and $\hat{\mathcal{L}}$ such that $\hat{\mathcal{L}}[\mathfrak{uid}] = \mathcal{L}[\mathfrak{uid}]$ (*i.e.,* the simulated pair records the same balance for all corrupted users as the real valid pair). This follows from π_{users} and π_{solvency} being zero-knowledge (Corollary 5 and 6), where the former covers the case that the universally verifiable proof reveals private information, and the latter covers the supplementary user check proof.

Therefore Xiezhi is a privacy-preserving proof of solvency.

D The Extension for Hashed Keys

In π_{keys}, we showed how to prove the knowledge of a private key and publish the corresponding selector value through the OR proof. However, it is ideal to allow the prover and the verifier to directly interact with hashed keys for simplicity and security. Agrawal *et al.* [1] introduced a way to prove an algebraic statement and a non-algebraic statement. In the context of proof of assets, the algebraic statement is the knowledge of the private key, and the non-algebraic statement is the correctness of the hash value of the public key. Note that the protocol in Agrawal *et al.* [1] is also a sigma protocol, which means it is natural to have Xiezhi support hashed keys as well. We describe the new π_{keys} with such extension in Protocol 8.

$\mathcal{P}$ and $\mathcal{V}$ are both given $\{\mathsf{addr}_i = \mathcal{H}(\mathsf{pk}_i), \mathbf{C}_{\mathtt{bls}}(s_i)\}$ where s_i is the i-th element in the selector vector $\overline{s} = \langle s_1, s_2, \ldots, s_n \rangle$, and an arithmetic circuit f such that

$$f(\mathsf{addr}_i, \mathsf{pk}_i) = \begin{cases} 1 \text{ if } \mathcal{H}(\mathsf{pk}_i) = \mathsf{addr}_i \\ 0 \text{ otherwise.} \end{cases}$$

$\mathcal{P}$ has the access to $\{\mathsf{sk}_i\}, \overline{s}$ and all hiding factors of relevant Pedersen commitments. Particularly, let r_i denote the hiding factor of $\mathbf{C}_{\mathtt{bls}}(s_i)$.

1. Case 1: $s_i = 1$ ($\mathcal{P}$ claims knowledge of sk_i)
   (a) $\mathcal{P}$ commits to $\mathsf{addr}_i, \mathsf{sk}_i, \mathsf{pk}_i$ and sends the commitments $comm_1 = \mathbf{C}_{\mathtt{secp}}(\mathsf{addr}_i), comm_2 = \mathbf{C}'_{\mathtt{secp}}(\mathsf{sk}_i)$ and $comm_3 = \mathbf{C}_{\mathtt{secp}}(\mathsf{pk}_i)$ where $\mathbf{C}'_{\mathtt{secp}}$ has a different generator from $\mathbf{C}_{\mathtt{secp}}$ and the discrete logarithm between these two is unknown to $\mathcal{P}$
   (b) $\mathcal{P}$ selects $e_1 \xleftarrow{\$} \{0,1\}^t; z_3 \xleftarrow{\$} \mathbb{Z}_{\mathtt{bls}}$
   (c) $\mathcal{P}$ publishes $t_3 = g_{\mathtt{bls}}^{-e_1} h_{\mathtt{bls}}^{z_3 - r_i e_1}$
   (d) $\mathcal{V}$ publishes t-bit challenge $e \xleftarrow{\$} \{0,1\}^t$ (or $\mathcal{P}$ via Fiat-Shamir)
   (e) $\mathcal{P}$ computes $e_0 = e \oplus e_1$ and publishes e_0 and e_1
   (f) Given the challenge e_0, $\mathcal{P}$ runs the protocols `ddlog` and the sigma protocol `comSigma` to prove the following statement: $\{(\mathsf{sk}_i, \mathsf{pk}_i) : \mathbf{C}_{\mathtt{secp}}(\mathsf{sk}_i) = \mathsf{pk}_i \wedge comm_2 = \mathbf{C}'_{\mathtt{secp}}(\mathsf{sk}_i) \wedge comm_3 = \mathbf{C}_{\mathtt{secp}}(\mathsf{pk}_i)\}$ (denote the proofs by π_1 and π_2, respectively)
   (g) $\mathcal{P}$ runs `comInSnark` to prove $f(\mathsf{addr}_i, \mathsf{pk}_i) = 1$ given $comm_1, comm_3$ (denote the proof by π_3)
   (h) $\mathcal{P}$ publishes the proofs π_1, π_2 and π_3
   (i) $\mathcal{P}$ publishes z_3
2. Case 2: $s_i = 0$ ($\mathcal{P}$ does not claim knowledge of sk_i)
   (a) $\mathcal{P}$ commits to $\mathsf{addr}_i, \mathsf{pk}_i$ and sends the commitments $comm_1 = \mathbf{C}_{\mathtt{secp}}(\mathsf{addr}_i), comm_3 = \mathbf{C}_{\mathtt{secp}}(\mathsf{pk}_i)$
   (b) $\mathcal{P}$ selects $e_0 \xleftarrow{\$} \{0,1\}^t; \alpha \xleftarrow{\$} \mathbb{Z}_{\mathtt{bls}}$
   (c) $\mathcal{P}$ runs the protocol `ddlog` and `comSigma` given the challenge e_0 in advance (The proofs π_1 and π_2 are always accepted by $\mathcal{V}$ as `ddlog` and `comSigma` are sigma protocols)
   (d) $\mathcal{P}$ runs `comInSnark` to prove $f(\mathsf{addr}_i, \mathsf{pk}_i) = 1$ given $comm_1, comm_3$ (denote the proof by π_3)
   (e) $\mathcal{P}$ publishes $t_3 = h_{\mathtt{bls}}^{\alpha}$
   (f) $\mathcal{V}$ publishes t-bit challenge $e \xleftarrow{\$} \{0,1\}^t$ (or $\mathcal{P}$ via Fiat-Shamir)
   (g) $\mathcal{P}$ computes $e_1 = e \oplus e_0$ and publishes e_0 and e_1
   (h) $\mathcal{P}$ publishes π_1, π_2, π_3
   (i) $\mathcal{P}$ publishes $z_3 = e_1 r_i + \alpha$
3. $\mathcal{V}$ outputs **acc** if and only if
   (a) $e = e_0 \oplus e_1$
   (b) $h_{\mathtt{bls}}^{z_3} = \mathbf{C}_{\mathtt{bls}}(s_i)^{e_1} t_3$
   (c) π_1, π_2, π_3 are valid

Protocol 8: The extension for π_{keys} to support hashed keys. The protocols `ddlog`, `comSigma`, `comInSnark` can be found in Section 3.1, 3.2, and 4.2 of Agrawal *et al.* [1], respectively.